N00167-76-M-8381
Item 0001AB

Working Paper No. 196-5
Aug[redacted]976

LEVEL II

WORKING PAPER.

ADA081566

ADVANCED NAVAL VEHICLE STRUCTURAL LOADS CRITERIA AND STRUCTURAL CHARACTERISTICS.

for

David W. Taylor Naval Ship Research & Development Center

Bethesda, Maryland 20084

Contract No. N00167-76-M-8381

Payne Inc.
1910 Forest Drive · Annapolis, Md. 21401

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER W.P. No. 196-5	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Advanced Naval Vehicle Structural Loads Criteria and Structural Characteristics (U)		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s)		8. CONTRACT OR GRANT NUMBER(s) N00167-76-M-8381
9. PERFORMING ORGANIZATION NAME AND ADDRESS Payne, Inc. Annapolis, Maryland 21401		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS CNO (OP96V) Washington, D.C. 20350		12. REPORT DATE August 1976 13. NUMBER OF PAGES 54
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) DTNSRDC Bethesda, Maryland 20084		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Unlimited and approved for Public release.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Advanced Naval Vehicle Concepts Evaluation
ANVCE
Technology Assessment
Advanced Naval Vehicle
Structural Loads Criteria
Structural Characteristics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper describes a method of determining structural design loads which is based on, what are now, generally accepted, probabilistic principles. The basic concept is that each ANVCE vehicle should have a reasonable probability of being capable of withstanding the most severe load which it may be expected to encounter during its normal operating life. The "reasonable probability" has been defined here in terms of factors of safety on yield, ultimate and fatigue loads.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

A

1. SUMMARY AND CONCLUSIONS

This paper describes a method of determining structural design loads which is based on, what are now, generally accepted, probabilistic principles. The basic concept is that each ANVCE vehicle should have a reasonable probability of being capable of withstanding the most severe load which it may be expected to encounter during its normal operating life. The "reasonable probability" has been defined here in terms of factors of safety on yield, ultimate and fatigue loads (see Table 2).

At first glance it may appear that the procedure described is too complex to be used in point design. For some cases where very little information exists this may be true. It is, however, the basic philosophy of the probabilistic approach that is important; this approach can be applied at any level of detail or, if necessary, approximation. Once the framework is established and the first calculations made it should become apparent which areas are critical to design and which require further experimental or analytical investigation.

Any rational, defensible procedure used to determine design loads under this basic concept is acceptable. A particular methodology has been outlined in this paper but others can be used equally well. Full-scale trial data, sub-scale tow-tank model data or analytical methods can all be adapted to the procedures described. The use of empirical formulae is discouraged unless the rationale behind the formulae can be interpreted in a compatible probabilistic format. The use of arbitrary, deterministic conditions is also discouraged such as the traditional procedure of supporting a ship on a wave of length equal to the ship waterline length and height to length ratio of 1/20, or the use of a 10 ft/sec sinking speed to define hydrofoil hull impact pressures. These conditions are meaningless without supporting analysis to determine how likely they are to occur and how often worse conditions might occur, in the particular environment to which the ANVCE vehicles will be exposed.

The object of the specified concept and specified factors of safety is not to enforce uniformity of approach but to promote the development of a set of comparable designs, developed to uniform standards.

2. INTRODUCTION

The concept of rational structural design involves the determination of loads on the basis of scientific rather than empirical procedures, in order that uncertainties may be reduced to a minimum. The concept is consistent with the modern approach to structural design that considers the "demand" upon and "capability" of the structure. In short, instead of insuring that a simple, calculated, design stress is below the ultimate strength of the material by an arbitrary factor of safety, an attempt is made to determine the demand of all loads acting on the structure and then the capability in terms of load-carrying ability -- the load the structure can withstand without failure. This approach requires a definition of failure, which may be a serious buckle, a major crack, complete collapse, or a tensile failure. The concept of rational design of a ship hull is believed to be consistent with a probabilistic approach, which has been found to be essential for dealing with random seaway loadings. Both demand and capability can be expressed in terms of probabilities, and a satisfactory design is then one in which the probability of failure is reduced to an acceptably low value (Reference 1).

The structural design criteria for an ANVCE candidate vehicle, therefore, should be based on the following three basic considerations:

1) The operational profile of the vehicle, which specifies how the vehicle is to be used in terms of loading, endurance, speed, transient maneuvers, mode of operation (cushion-borne, foil-borne, air-borne, hull-borne, etc.)

2) Analysis of the environment, which determines what weather conditions the vehicle will be likely to encounter in those areas in which it will be required to operate.

3) Definition of acceptable levels of structural reliability. In statistical terms this implies defining an acceptable return period* for the design limit loads, which defines the probability of exceeding the design structural load during the vehicle's lifetime, and, in conjunction with the selected, structural-strength distribution, determines the probability of structural failure during the vehicle's life.

Most of the vehicles represented in the ANVCE program are, by definition, of advanced design, so that rather little prior experience is normally available. Empirical formulae, based on prior vessel experience, has, traditionally, been the mainstay of naval architecture for very many years. It is contended here that empirical formula and arbitrary load conditions can only be accepted as a last resort for advanced marine vehicles. Instead, every effort should be made to determine loads in a rational, probabilistic manner that is consistent within itself and consistent with the operational expectation of the vehicle.

---

* Return period = $(\text{frequency of occurrence})^{-1}$

3. BASIC METHODOLOGY FOR DETERMINING STRUCTURAL LOADS

The loads experienced by an ocean-going craft depend upon a very large number of influences; they vary with time in a random manner; they never occur in exactly the same way twice. Some of the influences are vehicle-related such as speed, weight distribution and control settings and some are dependent on the environment such as wave height and wind speed. The structural designer has to convert this constantly changing scene to a limited number of design load cases which can be used to determine the craft's scantlings.

The method adopted here consists basically of the following steps:

- Analysis of the Operational Environment
- Selection of representative loading cases
- Determination of the probability of occurrence of each of the loading cases
- Calculation of the loads for each representative case in the form of an average or root mean square value together with a short-term probability distribution to describe the possible variations of loads under these conditions
- Computation of long-term probability distributions by combining the short-term distributions
- Selection of design loads.

In order to define the problem therefore, the first requirement is to define the operational environment.

Operational Environment

Figure 1 shows a typical operational envelope for an advanced marine vehicle, in this case an early version of an amphibious assault landing craft. The craft is expected to operate within the speed wave-height envelope shown. The proportion of time spent in each area of the envelope is indicated. The distribution on the wave-height axis is derived directly from ocean wave-height data as suggested by Figure 2. The horizontal distribution along the speed scale was derived from a typical mission profile such as that shown in Figure 3. The sloping lines dividing the speed zones represent the fact that the craft's speed must fall off as sea state increases. This particular craft, for example, could reach 90 knots in calm water but was not expected to exceed 40 knots in SS4. It was not expected to operate on-cushion in sea states higher than SS4. The loads experienced in this case in SS4 even at 40 knots were so severe that consideration was given to reducing speed still more in the higher sea states. This consideration is represented by the two lines labeled "structural loads limitations". One point of interest in this diagram is that the percentage of time spent in the most severe conditions is really quite small. 2.2% of the total time is spent in SS4, between wave heights ($H_{1/10}$) of 5.8 and 8.7 feet and between speeds of 25 and 48 knots. However, only a small proportion of this

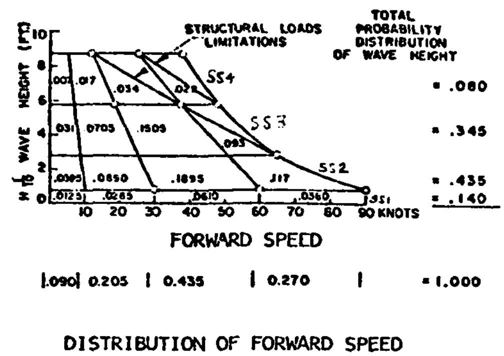


Figure 1. Typical Operating Envelope for Advanced Marine Vehicles.

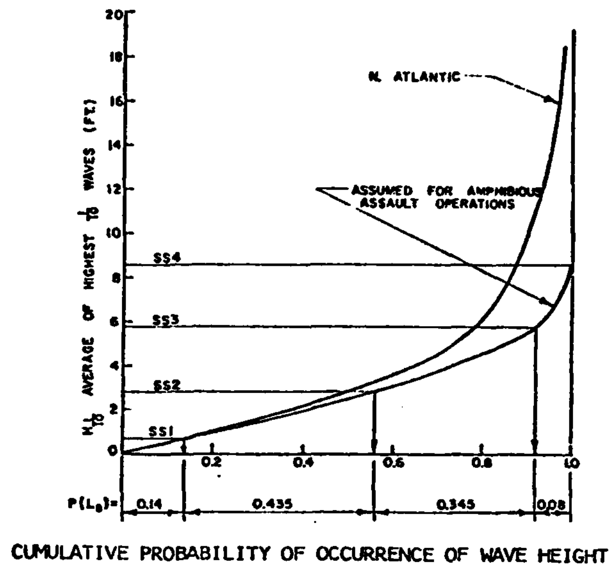


Figure 2. Sea State Probabilities for Typical Amphibious Assault Mission. (Ref. 8)

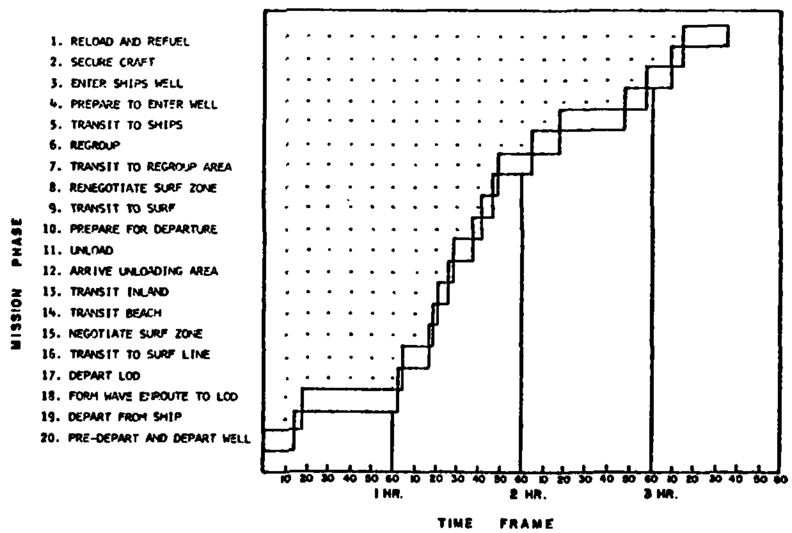


Figure 3. Analysis of Mission Profile for Amphibious Assault Landing Craft. (Ref. 8)

2.2% is actually spent near the extreme corner of the diagram (at 40 knots in wave heights of 8.7 feet). If this distribution within each area is not taken into account, the results can be misleading. In the case shown in Figure 1, the cases actually used for computation are shown by the small open circles.

Speed and sea state are not the only quantities that define the operational environment, however; it is equally important to define the distribution of heading angle to the dominant wave direction and also distribution of gross weight. Typical distributions of these quantities are shown in Figs. 4 & 5. All headings are normally assumed to be equally probable and a gross weight distribution can usually be derived by a brief analysis of the typical mission profiles or the top-level requirements.

By means of the foregoing considerations, therefore, a matrix of representative cases has been developed. For each case, a set of loads is required, which can be generated either by computational methods or by making use of experimental results. In practice, a combination of both sources is often used. For each condition, the designer is interested in a wide variety of load information.

Types of Loads

The designer requires information on a large number of types of loads such as the following:

- Midship vertical bending moment and shear force
- Longitudinal distribution of bending moment
- Transverse bending moments
- Torsional moments
- Hydrodynamic and hydrostatic pressure distributions.

In all cases, information will be required on limit load and fatigue loads. These loads may be due to a variety of causes which are normally superimposed on each other. The most significant causes of load are:

- Still-water loads - due to the interaction of buoyancy, lift forces and weight distribution
- Thermal stresses - due to the different temperatures existing at different components of the structure
- Wave-induced loads - which are normally the most significant as far as design limit loads are concerned
- Slam-induced loads - which occur when the hard-structure of an ACV or the hull of a hydrofoil hits the water. Conventional ships and planing craft are also subjected to slamming loads on their bow and bottom areas. These loads are normally superimposed on the wave-induced loads.

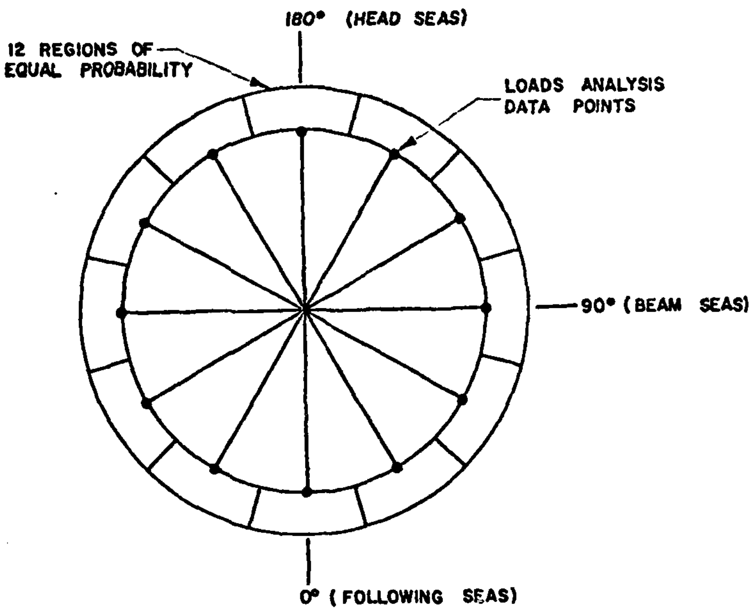


Figure 4. Assumed Distribution of Ship Heading to Wave Direction.

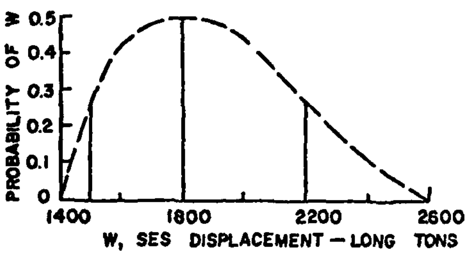


Figure 5. Assumed Distribution of Gross Weight.

An illustration of the relative significance of different load types during a single voyage of a merchant ship is shown in Figure 6 (Reference 1).

A procedure is required to consider each of these loads in turn, estimate their magnitudes and probability distributions and to determine to what extent they should be superimposed on each other. The method described here attempts to carry this out. It is inevitable that some of the steps in the method are considerably weaker than others, but the important thing is that it does produce usable numbers for the designer. As time goes on, each step can be re-examined and the validity of the total method can be steadily improved. In order to determine these loads, however, hard data is required on each and every one of the selected cases. The sources of data will be discussed in the next section.

Sources of Information

As more and more research and development work is done on an ever-increasing number and variety of advanced marine vehicles, a larger and larger data base becomes available. The advanced marine vehicle community have, perhaps, followed the lead of the conventional ship designers (Refs. 1, 2, 3, etc.) in attempting to replace empirical formulae by rational design procedures, and by extensive full-scale experimentation.

Available sources of information can perhaps be divided into three main groups:

- Analytical load prediction procedures
- Model test results
- Full-scale test results.

These will be discussed, in turn, below.

Analytical Load Prediction Procedures:

Empirical Formulae -

The simplest form of analytical procedure is the empirical formula. It should not be forgotten that until less than twenty years ago, many successful ships and sea planes were designed to formulae (Reference 4, for example). Before the advent of modern computational techniques this was often the only procedure available to most designers.

Frequency-Domain Simulation -

Provided that the response of the vehicle to a seaway can be assumed to be linear, then frequency-domain analysis can provide load and motion information in a reasonably economical fashion. This procedure is described in Reference 5, for example. The response amplitude operators for the quantity in question (midship bending moment, pitch motion, etc.) can be computed by treating the hydrodynamic and hydrostatic forces acting on the vehicle as linear systems. The root mean square (rms) response of each quantity can then be determined for any specific condition of operation.

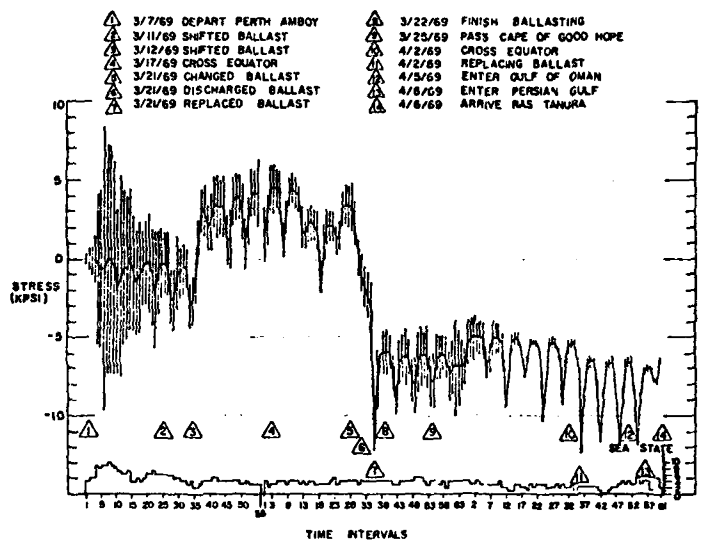


Figure 6. Typical Variation of Midship Vertical Bending Stress During a Voyage of the S.S.R.G.FOLLIS. (Reference 1)

In spite of the fact that most advanced ship response systems are very obviously nonlinear, it has been found, experimentally, that the agreement between rms responses measured in the model tank and those predicted by frequency-domain simulation is surprisingly good, even for operation in severe sea states. (References 6 and 7). It should be emphasized, however, that it is, often, only the rms values that correspond to the predicted values. As the responses are, typically, nonlinear, especially in high sea states, the distributions of peak responses do not follow the Rayleigh distribution and the maximum values predicted by using the Rayleigh distribution constants do not, in general, match experimental values.

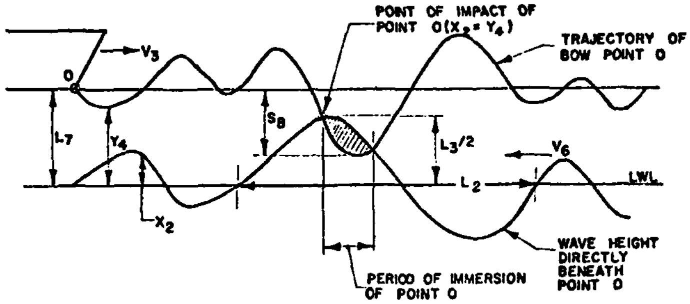
The great advantage of the frequency-domain simulation is that it is simple, flexible and inexpensive. As it has now been proven, many times, to yield usable values of rms response, it provides a very valuable design tool.

Time-Domain Simulation -

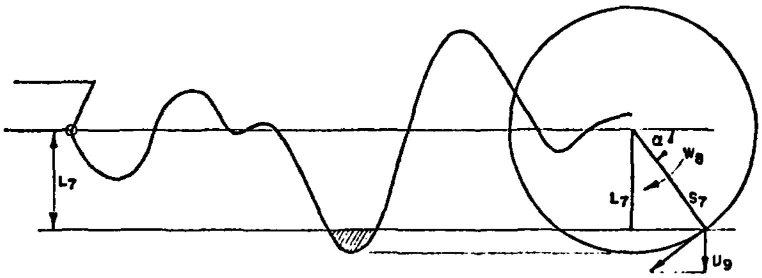
A great deal of effort has been devoted to developing very comprehensive time-domain simulations of advanced marine vehicles. For example, Reference 8 describes a simulation of the SES which has been under more-or-less continuous development ever since 1970. This particular model provides a detailed representation of the SES that may be used to generate six-degree-of-freedom response to wave action, maneuverability characteristics, simulated emergency conditions, etc. As time goes on, and the power of computation techniques increase, it is very probable that the time-domain approach will be used more and more in design work but at the present state-of-the-art, it does have some disadvantages. Firstly, it is a very elaborate procedure to use, so that when engineering time and computer hours are taken into account, it is very expensive to operate. Secondly, due to the expense involved, it is not usually possible to run the program for long enough to obtain a reasonable statistical sample. By its nature, the time-domain model is deterministic whereas the real world of wave-induced loads (even in the model tank) is probabilistic. The current tendency seems to be to use time-domain models for specific, time-constrained, transient events, such as maneuvering and emergency ditching, etc., rather than for steady-state operation.

Impact Simulation -

A much less ambitious time-domain simulation, that has proven itself useful for many years, is the impact program which has been used in different versions, to generate impact load time histories for the SES-100A (Reference 9), the JEFF-A Landing Craft (Reference 10), the Arctic SEV study (Reference 11) and the Litton and Rohr 2KSES designs. The impact analysis makes use of the assumption that the normal operation of an advanced marine vehicle, when cushion-borne or foil-borne, is more-or-less impact-free, so that impacts of the hard-structure of the hull or wet-deck on the water surface are comparatively rare events. It follows that the motion of the vehicle prior to the slam can be predicted satisfactorily by a frequency-domain simulation. By using the characteristic parameters of the Rayleigh (or Weibull or generalized gamma, etc.) distribution, the probabilities of a slam occurring at any point on the craft at any particular velocity can be determined. A slam event is illustrated by Figure 7. The impact program logic converts the probabilistic information of the frequency-domain program into a deterministic scenario represented by Figure 8. Wave



(a) Trajectories of bow and local wave height relative to level water line.



(b) Trajectory of bow relative to local wave height.

Figure 7. Diagram Showing Impact Event Occurring on a Typical SES Bow. (Reference 9).

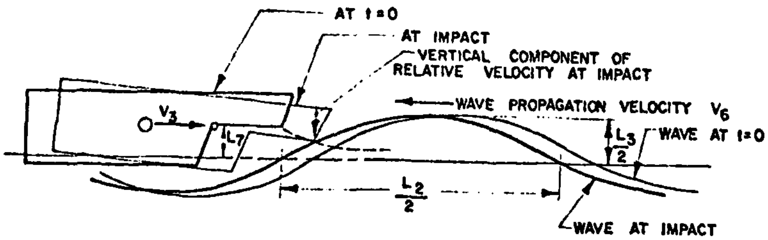


Figure 8. Diagram Representing Bow Impact of a Typical SES. (Reference 9).

heights and lengths are chosen to be compatible with the frequency-domain information and other parameters are chosen to provide representative situations. Considerable effort has been expended, over the years, in studying the sensitivity of the final results to the assumptions made. When necessary, ranges of values have been used to provide multiple simulations. By treating each element of the wetted impact area as a section of an impacting wedge, defined by effective trim and deadrise angles, and by applying modified versions of the techniques developed in References 12, 13, 14 and 15, it is possible to generate a pressure map of the impact area. Integration of the pressure yields total load at each instant of time and, consequently, the vehicle response can be calculated. A typical time-history is illustrated in Figure 9.

By carrying out a number of these impact calculations, each of which is associated with a specific frequency of occurrence, it is possible to generate a long-term, probabilistic picture of the impact loads that any vehicle may experience during its lifetime. This procedure will be covered in a later section.

Model Tests

A number of structural model test techniques have been developed over the last few years and the application of these techniques to the advanced marine vehicle field has yielded a very significant amount of useful data (References 6 and 7 for example).

Segmented models have been instrumented to provide measures of shear force and bending moments at one or two hull stations. These models have been tested in head seas and in impact drop tests. These models are similar to those that have been used in tests of conventional ship models (Reference 16).

A rigid, vinyl modelling technique has been developed at DTNSRDC which allows modelling of structural response as well as structural loads. This is achieved by using the fact that the elastic modulus of the rigid vinyl is such that its elastic properties scale properly for 1/30-scale models of aluminum prototypes or 1/28-scale of steel prototypes (References 17 and 18). This method was used to build a model of the CPIC which was tested at DTNSRDC.

A similar technique has also been developed in which scale models have been built by using a segmented model whose component segments are attached to a carefully designed scaled structural grillage. This method has been very successful in measuring response of complex models in head and oblique seas. (References 19 and 20).

In the same way as in the field of full-scale testing, the amount of experimental model data available has changed abruptly from the situation a few years ago when almost nothing was available to the present situation where there is almost an embarrassing amount of data and insufficient time to analyze it all.

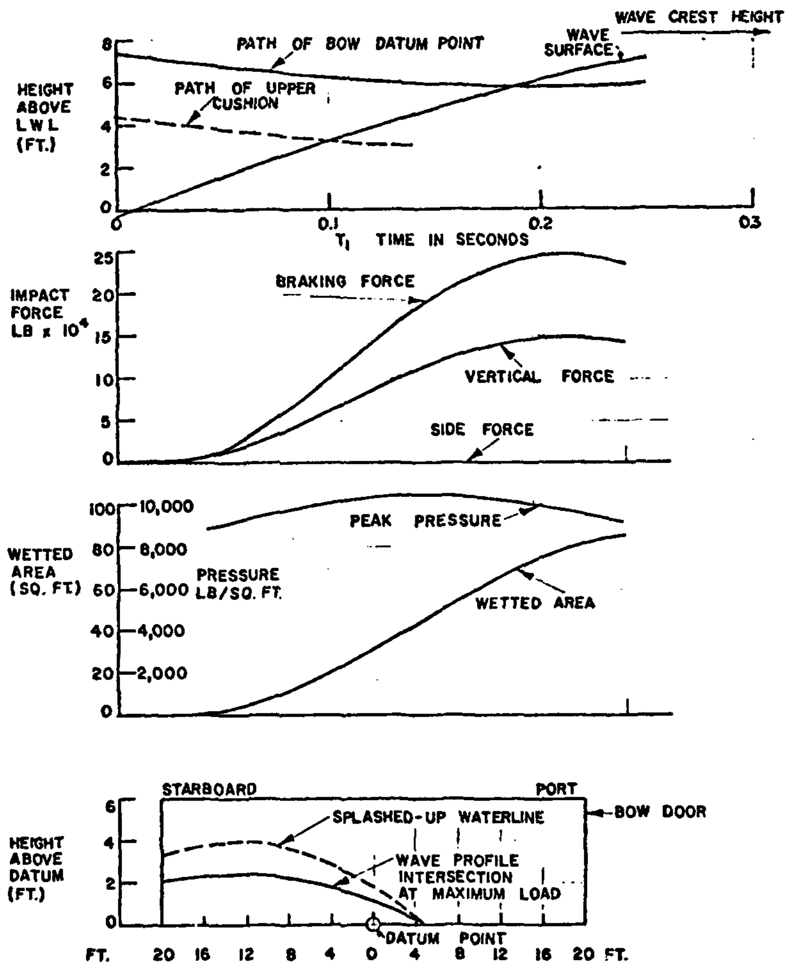


Figure 9. Time-History of an AALC Impact in SS4 at 38 Knots at 45° Heading to Waves. (Reference 10)

Full-Scale Trials

In the last few years, particularly in the SES field, the situation with regard to full-scale structural loads data has also improved dramatically. The SES-100A (Ref. 21) and the -100B (Ref. 22) and Ref. 6, after overcoming the usual number of teething problems, have both generated large quantities of structurally relevant data, as have the hydrofoil program, the subscale SES, XR1-D and the CPIC coastal patrol planing craft. For example, Figure 10 shown vertical accelerations measured on the SES-100A when running, on-cushion, in head seas. The asymmetric nature of the data is at once apparent. The peak upward accelerations tend to be larger than the down. Figure 11 shows similar data taken on the SES-100B plotted on Weibull probability paper. Straight lines on this paper represent cumulative Weibull distributions. The cumulative Weibull distribution function is given by:

$$P(x) = \exp\left[-(\lambda x)^{c}\right] \tag{1}$$

where λ is a constant and the index, c, is numerically equal to the gradient of the straight lines on Figure 11. If the index, c, (gradient) is 1.0, then the distribution is exponential, and if c is 2.0, it is a Rayleigh distribution as both of these distributions are special cases of the Weibull distribution. The experimental points lie consistently between these two extremes. In Figure 12, the corresponding trough accelerations are plotted, showing that, in the head-sea case, at least, a considerable difference exists between trends of peak and trough accelerations. This is consistent with Figure 10. Figure 13 shows plots of bending stress which exhibit very much the same characteristics as the acceleration data. The great advantage of full-scale testing is that the test runs can be several times longer than is possible in sub-scale model testing in towing tanks, so that the statistical samples obtained are much larger. Disadvantages of full-scale testing are that wave heights and other test conditions cannot be known very precisely and may vary considerably even during quite a short run. The full-scale tests, however, are run in those conditions for which prototype vehicles must be designed, so that a degree of realism is achieved that cannot be achieved in the model tank and the problems of scaling from, say, a 100T SES to a 2000T SES are very much less than those that were involved in scaling from an 8-foot model to the 100T vehicle.

Prediction of Full-Scale Loads

The structural designer requires specific information so that he can design his vehicle. He would like to know the largest midship bending moment, for example, to which his ship will be subjected within a twenty-year operational life. The hydrodynamicist cannot provide him with this deterministic figure. Instead, however, he can provide the structural designer with a bending moment which has only a ten percent, or one percent, probability of being exceeded during the lifetime of the ship. Once the probabilistic criteria (in this case the ship's life and the acceptable probability of exceedance) have been specified, a single, limit, design, bending moment can be calculated.

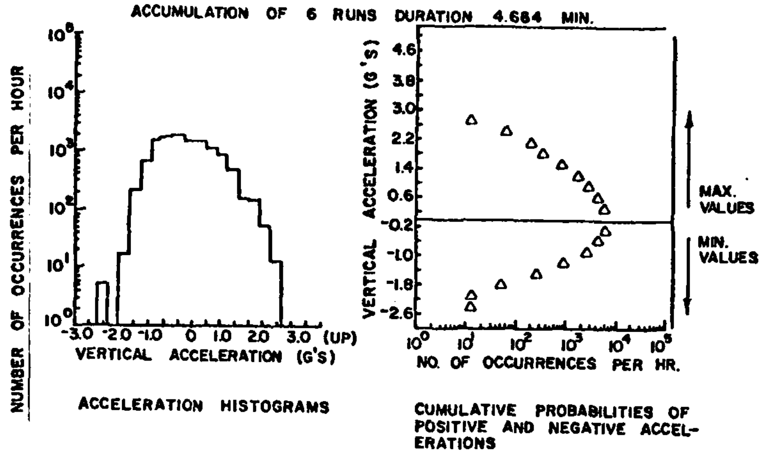


Figure 10. SES-100A Vertical Acceleration at C.G. (Reference 7)

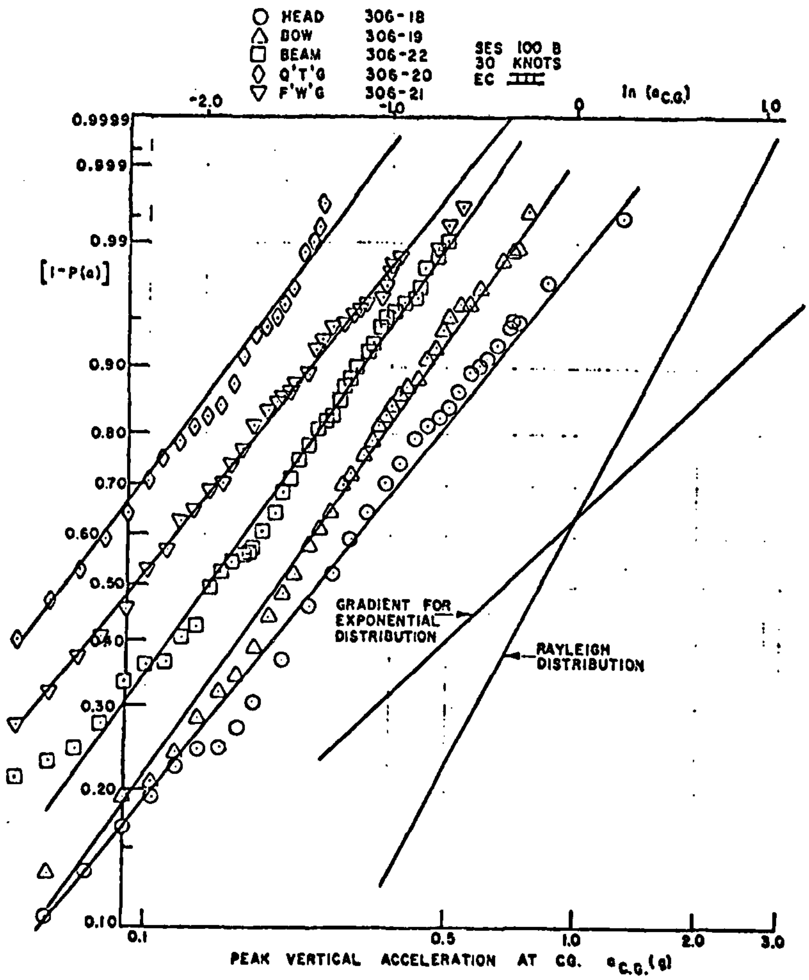


Figure 11. SES-100B Peak (Upward) Vertical Accelerations Measured at C.G. (Reference 7)

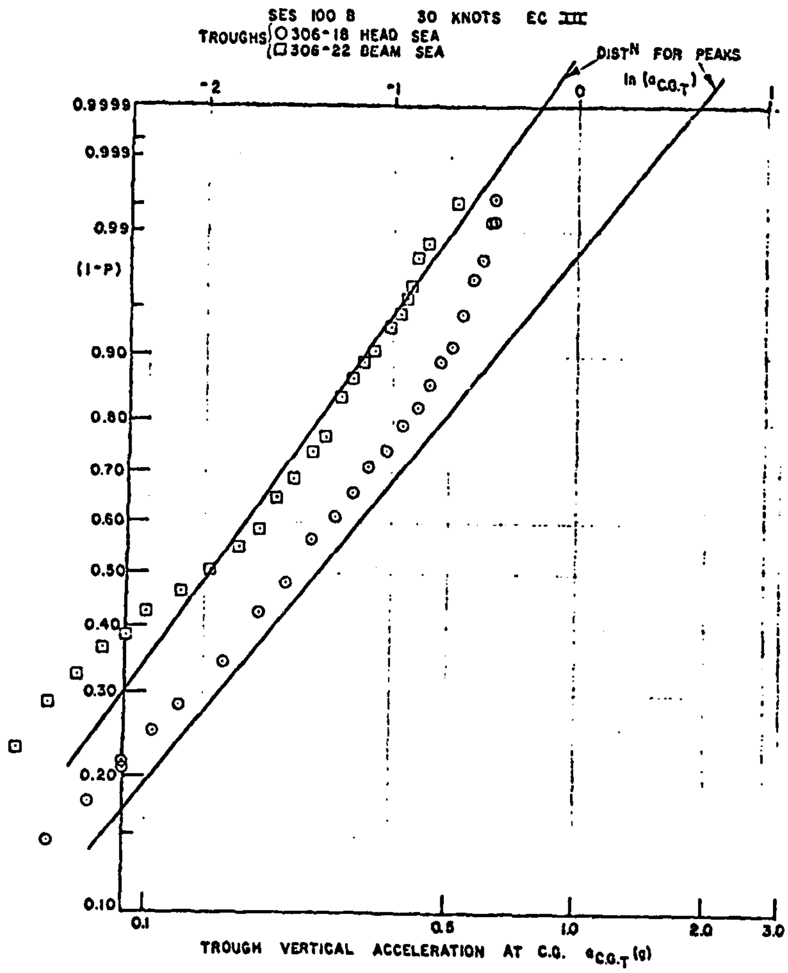


Figure 12. SES-100B Measured Trough (Downward) Acceleration Data. (Reference 7)

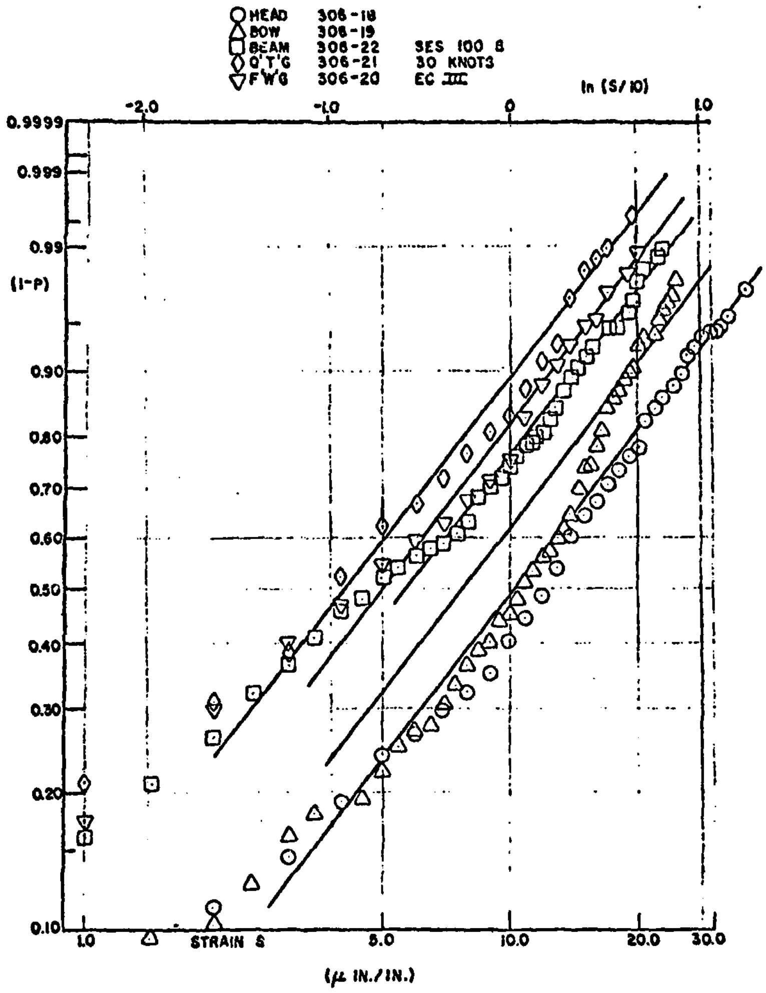


Figure 13. SES-100B Measured Peak Bending Stress Data. (Reference 7)

The methods of calculation vary. In broad terms, there are two schools of thought:

- One method is to design for the "worst expected sea condition". This method proposes to design the ship by analyzing the ship's response to the "worst storm" that it is likely to encounter. It is well known that the waves encountered in severe storms are very dissimilar to those of lesser storms. The duration of extreme storms is normally so limited that the seas do not have time to become anywhere near fully developed. Table 1 and Figure 14 illustrate this. The time required to "fully develop" the seas corresponding to the higher sea states exceeds the probable time that that sea state exists in one year. In this method, therefore, the design loads are determined by exposing the ship, analytically or experimentally, to seas corresponding to the worst storm records available. The main drawback to this method is that the available records of such storms are very limited in number, that the statistical sample used as a base is thus very limited and, in any case, as illustrated in Figure 15 (Ref. 25), recorded data shows that the maximum loads probably do not necessarily occur in the worst sea states.

- The second method is the statistical method in which extreme loads are predicted by extrapolating from a larger data base using all available sea state data. The weakness of this method is that there is no guarantee that the extrapolations are valid. Historical evidence, however, tends to support this procedure. The long-term predictions based on the first few instrumented voyages of the WOLVERINE STATE, for example, have, in general, tended to be borne out very well by data subsequently accumulated (Reference 3). It may be argued that a conventional ship is basically a linear system, whereas most advanced vehicles are not, but it is suggested here that, provided suitably flexible probability distributions are used to describe the available data, the nonlinearities do not affect the validity of the method. This method, at least, allows predictions to be made from data that is generally available or obtainable, and is flexible enough to allow refinement as time goes on. It can also make use of the data provided by the extreme sea state cases and provides a method for properly ordering and weighting such contributions.

Short-Term Statistics

In order to predict extreme values of loads or motions, it is necessary, therefore, to be able to extrapolate from available samples of experimental or analytical samples or experimental or analytical evidence. In general, the experimental samples are very small compared with the expected life of the vehicle, so that the extrapolation process must be done with considerable care. Figures 11, 12 and 13 show typical peak value distributions from the SES-100B experience in environmental condition EC III.* The distribution used here to approximate the data is the Weibull distribution, which is a special, two-parameter case of the three-parameter, generalized gamma distribution. SES structural models have been shown to exhibit reassuringly similar tendencies (Reference 16).

---

* Environmental conditions EC I, II and III are defined in Ref. 21 as implying significant wave heights of 0, 2.4 and 4.8 feet, respectively.

SS	0	1	2	3	4	5	6	7	8	9
$h_{1/3}$ (FT)	.018	1.2	3.1	5.3	7.4	12.5	20	42	61	">128"
WIND SPEED (KN)	3.5	9	13.5	17	19.5	24	29	39	45	>64
AVERAGE PERIOD T (SECS)	1.4	2.6	3.9	4.8	5.5	6.9	8.2	11.0	12.8	>19
$Q(h_{1/3})$			.83	.55	.30	.065	.02	.0003	7.3×10^{-6}	2.2×10^{-11}
PERCENTAGE OF TIME (N. ATLANTIC)		17%		28%	25%	21.5%	6.5%	1.97%	.0293%	.0007%
TIME IN ONE YEAR (HRS.)		1489		2453	2190	1893	569	173	26	3.7 MINUTES
NUMBER OF WAVES/YR. (MILLIONS)				2.1	1.5	1.1	.27	.065	.0079	14×10^{-6}
AVERAGE HIGHEST EXPECTED WAVE (FT.)		3.2		11.3	16.8	26.33	40.8	73.0	108	107
CONDITIONS FOR "FULLY DEVELOPED SEA"										
DURATION (HRS.)		2.2	5.0	7.0	9.5	14	21.5	40	103	490
FETCH (N.M.)		9	27	47	70	130	255	655	2700	50,000

Table 1. Theoretical Sea State Characteristics. (Reference 24)

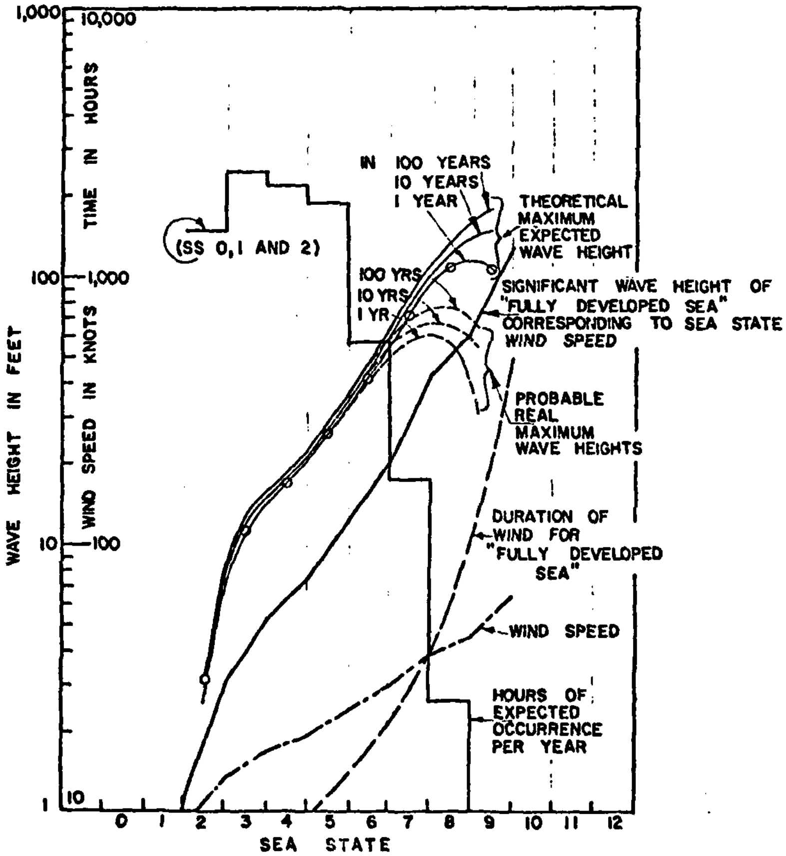


Figure 14. Theoretical Sea State Characteristics. (Reference 6)

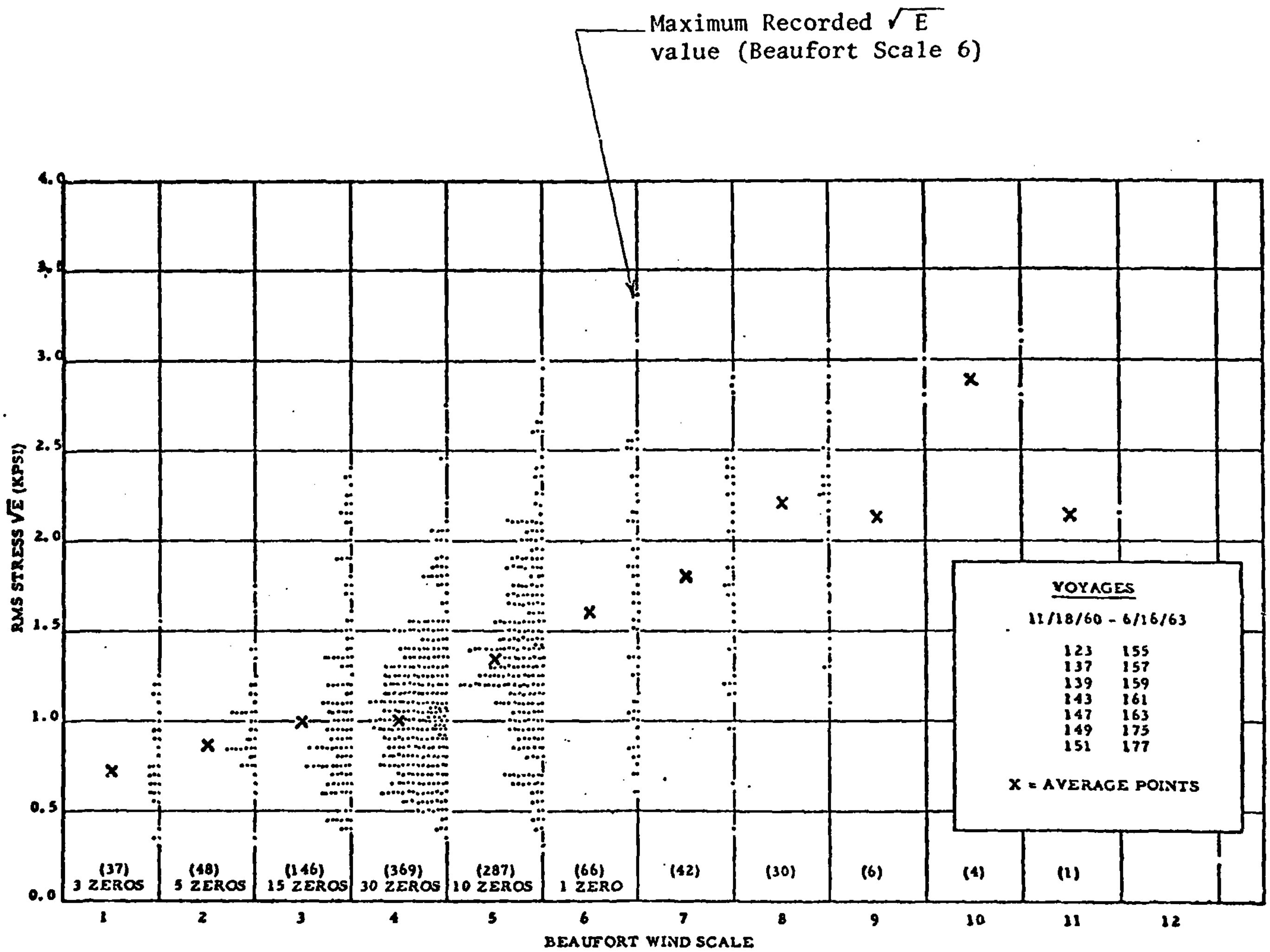


Figure 15. Midship Bending Stresses Measured on S.S.HOOSIER STATE. Each Dot represents the RMS of Peak-To-Peak Stresses During a Twenty Minute Sample Taken Every Four Hours for Fourteen Voyages. (Reference 25)

The effect of the choice of distribution on the long-term loads, moments and stresses may be seen most graphically in Figure 16, which shows the long-term distributions of x/x_{rms} for various c values. At the higher levels of probability, which are those of interest in establishing extreme values, the long-term cumulative distribution curves fan out over a wide range for different values of the Weibull parameter c.

On the basis of the evidence in Figures 11 - 13, it is suggested that the Weibull distribution may be said to represent the data in a reasonable manner, and that it is a clear improvement on either the exponential or the Rayleigh distributions which are, in turn, one-parameter special cases of the Weibull distribution (see Figure 11). The Weibull distribution is also acceptable from the point of view that it is closely related to the Rayleigh distribution which is generally accepted as representative of wave-heights and wave-induced phenomena. The theory of extremes (Reference 26) from which the first asymptotic distribution is derived is not really applicable to this type of data. The theory of extremes should be applied to the distribution of single extreme values from unrelated events (for example, the highest flood tide in each year or the highest single stress measured in each of a set of different storms or test runs). It should not be applied, therefore, to a series of peak values from a single continuous record, which is the extent of the majority of the data presently available. If a large enough number of runs were made on the manned models, it would be very relevant to examine the maximum values of each run by means of the statistics of extremes. This has not yet been attempted as the number of rough-water test runs on the manned models is still too limited.

In order to make use of the Weibull distribution in the analysis of extreme values, some of its characteristics must be understood. These are presented in Appendix A.

Long-Term Statistics

Each run in the test tank or of the full-scale craft is necessarily a short-term event. During each short-term event, it is assumed that the ship's speed and heading and the sea state remain constant. The operational life of a marine vehicle can be considered as a summation of a very large number of short-term events. If the behavior in each short-term event is known, and also if the manner of distribution of short-term events throughout the vehicle's life is known, then a long-term picture of the vehicle's life can be built up.

This is the method used here to predict the long-term behavior of the vehicle so that eventually, a single long-term distribution of each applied load or stress can be built up to represent the vehicle's total experience in all speeds, sea states, headings and loading conditions. In order to accomplish this, use is made of the description of the operational environment given earlier.

If the probability of exceeding a given wave-induced load x, at a speed V, in sea state S, at heading H and gross weight W, is $P_{VHSW}(x)$, then the total probability P(x) of exceeding the load x is given by summing the component probabilities for all conditions.

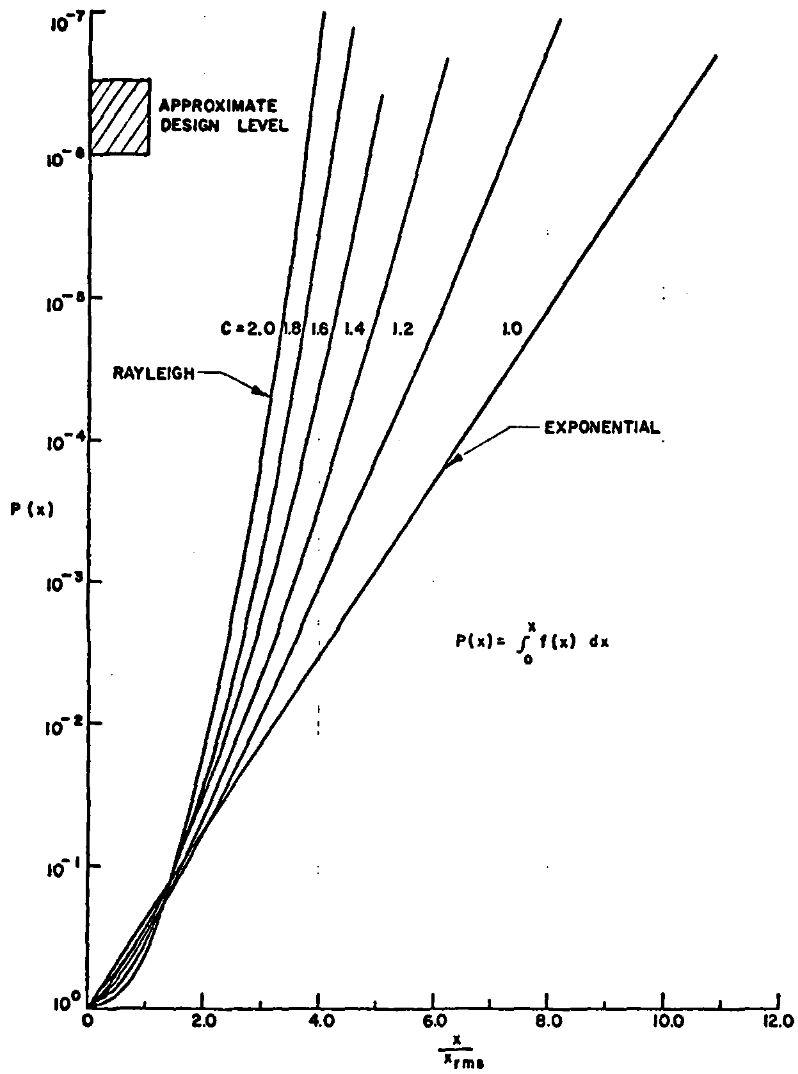


Figure 16. Cumulative Weibull Probability Distributions for a Range of Values of the Index c.

$$P(x) = \sum_V \sum_H \sum_S \sum_W P_V \cdot P_H \cdot P_S \cdot P_W \cdot P_{VHSW}(x) \tag{2}$$

where P_V, P_H, P_S, P_W are the probabilities of occurrence of each velocity, heading, sea state and gross weight.

Combined loads can be treated in the same way. Suppose, for example, that the probable occurrence of a still-water load x_{SW_i} (or thermal lcad, or etc.) is $P_i(x_{SW_i})$, and that this distribution is independent of the wave-induced load, then the total combined probability $P_c(x)$ of exceeding the same load x is given by:

$$P_c(x) = \sum_i P_i(x_{SW_i}) \; P(x - x_{SW_i}) \tag{3}$$

where $P(x - x_{SW_i})$ is defined by Equation (2).

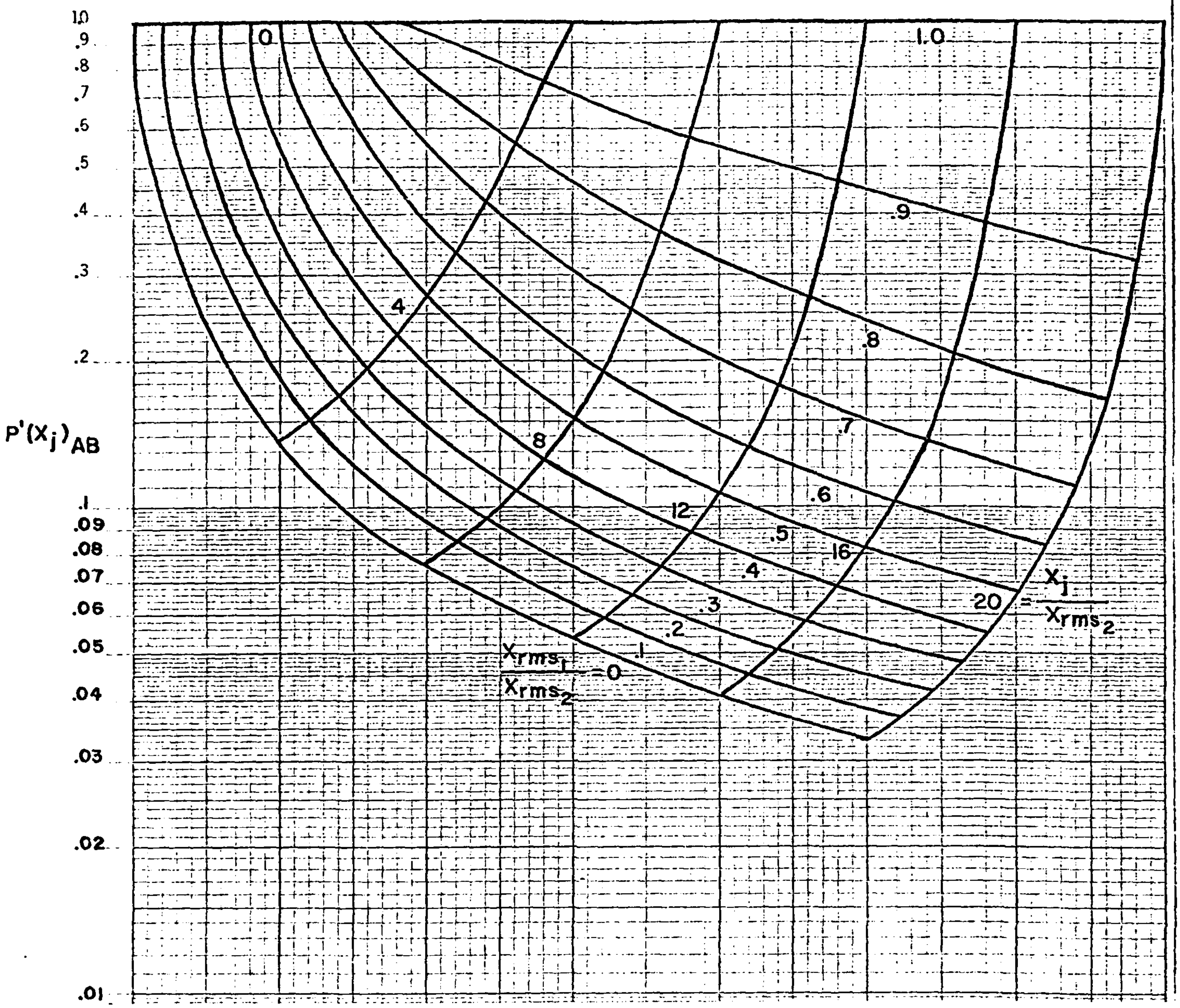
In fact, neither still-water loads nor thermal loads are normally independent of wave-induced loads, but this only provides a slight complication that can readily be resolved by computational procedures.

By basing the short-term extrapolations on experimentally measured stresses, the effects of the combined components of wave-induced loads are taken into account. The complex combinations of vertical and transverse bending moment and torsion can be represented in the model and full-scale tests in an adequate manner.

The number of cases selected from the operational envelope shown in Figure 1 is limited by the test and computational time available. As mentioned before, it may be misleadingly conservative to use the points at the top right-hand corner of each section of the operational envelope to define the conditions pertaining within that segment. Instead, it is often assumed that the distribution within each segment is uniform and a correction is applied, based on this assumption. Figure 17, for example, shows the effect on the long-term probability of exceeding a value x_j of a uniform spread of short-term Weibull distributions with rms values ranging from x_{rms_2} (upper limit) to x_{rms_1} (lower limit). The larger the value of x, and the smaller the ratio x_{rms_1}/x_{rms_2}, the greater the effect on the total probability. The ordinate $P'(x_j)_{AB}$ in Figure 17 is the ratio of the probability of exceeding x_j due to the uniformly distributed set of distributions to the probability of exceeding x_j due to a single distribution at x_{rms_2}.

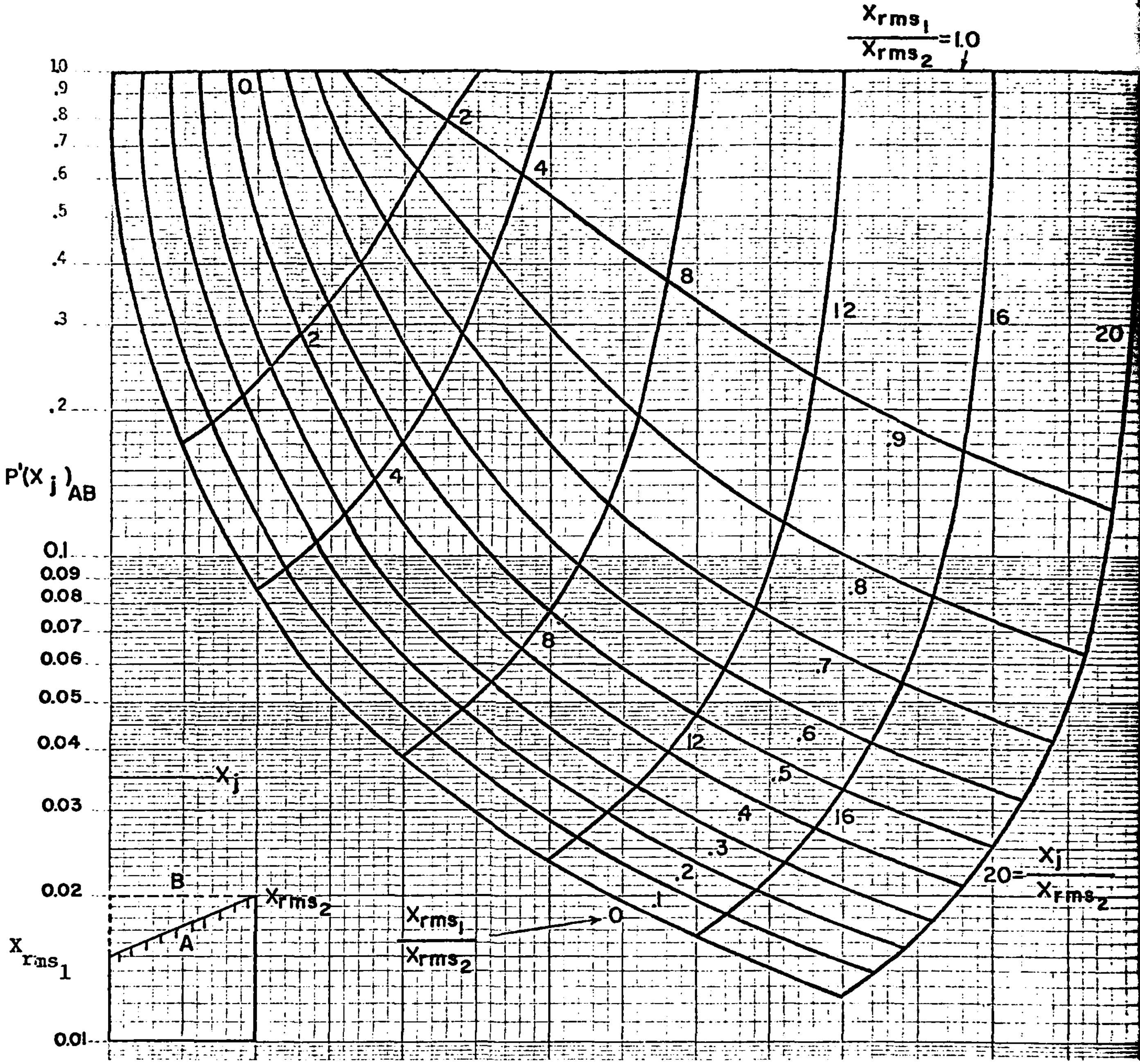
Selection of Design Limit Loads

By using the summation and integration procedures outlined in the previous sections, it is possible to generate a single, long-term probability distribution of each type of load or stress such as the familiar curves illustrated in Figure 18. The abscissa is a logarithmic scale that may represent probability of exceedance in each stress cycle, or number of cycles per return period, which may be readily converted to ship life in years. The left-hand end of this curve is relevant to rarely occurring limit loads and the right-hand end to multiple-cycle fatigue loads.



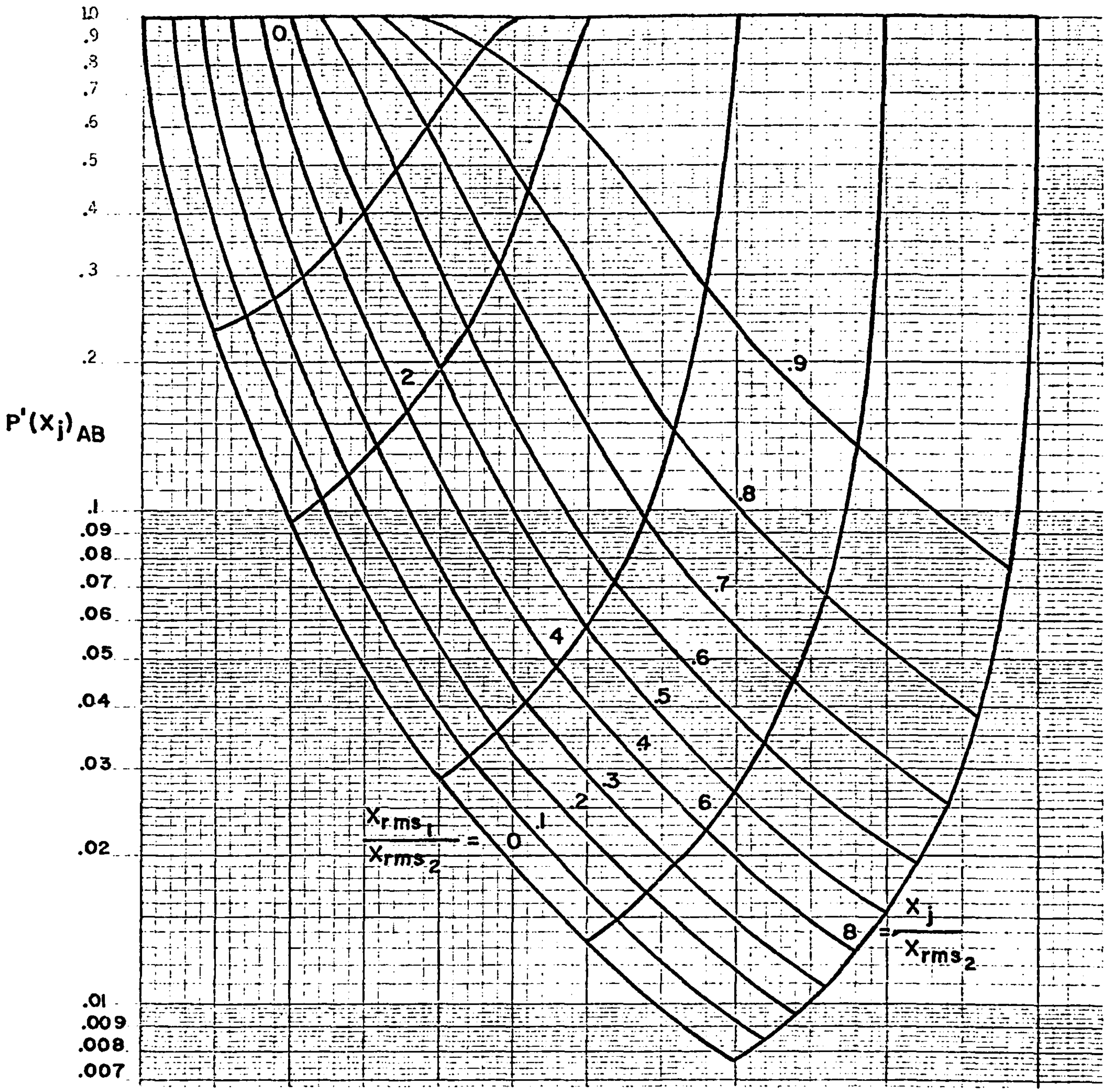
$P'(X_j)_{AB}$ is the ratio of the probability of exceeding X_j due to a process defined by a uniform "spread" of r.m.s. values from X_{RMS_1} to X_{RMS_2} to the probability of exceeding X_j due to a constant value X_{RMS_2}

Figure 17(a). Cumulative Distribution Characteristics of the Weibull Distribution. (C = 1.0). (Reference 6).



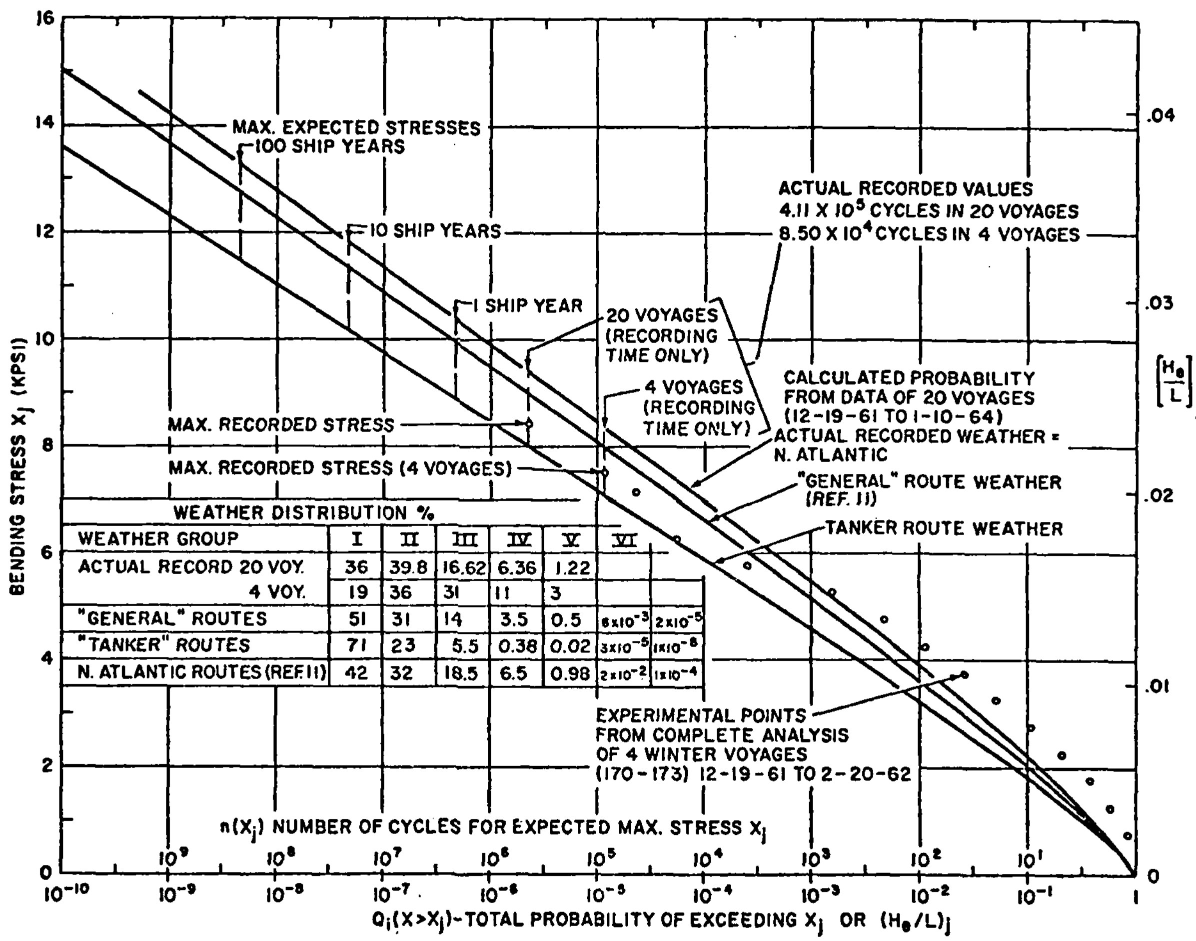
$P'(X_j)_{AB}$ is the ratio of the probability of exceeding X_j due to a process defined by a uniform "spread" of r.m.s. values from X_{RMS_1} to X_{RMS_2} to the probability of exceeding X_j due to a constant value X_{RMS_2}

Figure 17(b). Cumulative Distribution Characteristics of the Weibull Distribution. (C = 1.3). (Reference 6).



$P'(X_j)_{AB}$ is the ratio of the probability of exceeding X_j due to a process defined by a uniform "spread" of r.m.s. values from X_{RMS_1} to X_{RMS_2} to the probability of exceeding X_j due to a constant value X_{RMS_2}.

Figure 17(c). Cumulative Distribution Characteristics of the Weibull Distribution. (C = 2.0). (Reference 6).



WEATHER DISTRIBUTION %							
WEATHER GROUP	I	II	III	IV	V	VI	
ACTUAL RECORD 20 VOY.	36	39.8	16.62	6.36	1.22		
4 VOY.	19	36	31	11	3		
"GENERAL" ROUTES	51	31	14	3.5	0.5	6x10^-3	2x10^-5
"TANKER" ROUTES	71	23	5.5	0.38	0.02	3x10^-5	1x10^-8
N. ATLANTIC ROUTES (REF. 11)	42	32	18.5	6.5	0.98	2x10^-2	1x10^-4

Figure 18. Effect of Weather Distribution on Long-Term Bending Moment Probability, S.S. WOLVERINE STATE. (Reference 25)

The design limit load may be read off this curve at one, ten or one-hundred ship lives* It is simple to show that by selecting one-hundred ship lives as the return period, then each ship has a one-percent chance of exceeding this design load during its operational life. Fortunately, the increase in stress level between one, ten or one-hundred lives is, normally, not very large so that a conservative choice may be made without too great a penalty. The sensitivity of a typical, design, SES bending moment to some of the principal parameters involved is shown in Figures 19, 20 and 21. In Figure 19, the effect of operational life is seen to be rather small compared with the effect of the Weibull parameter c. Figure 20 shows the effect of limiting the maximum on-cushion sea state and Figure 21 shows the slight alleviation that can be achieved by allowing a deviation in heading in severe sea states. In all cases, the Weibull parameter, c, is seen to dominate the result so that considerable care should be devoted to its proper evaluation.

Structural Fatigue

Cyclic fatigue loads will occur in a number of situations and in a number of different elements of the structure. The principal causes of repetitive cyclic loading are the following:

- Still-water bending loads varying once per voyage, or per refuelling cycle due to changes in payload, stores, fuel load or ballast
- Diurnal thermal stresses, due to changes of air temperature from day to night and from sunlight to shade
- Occasional severe slamming loads which may occur every few minutes under storm conditions
- Wave bending loads which will occur at wave encounter frequency
- Vibrational loads caused by structural response to slamming ("whipping"), wave-induced loads ("springing") or by machinery-induced vibration
- Flow-induced vibration due to turbulence or cavitation of immersed appendages.

A single slam may result in a large number of stress reversals due to the whipping response of the structure. While little data is available for high performance ships, a great deal of data exists for conventional ships. Figure 22 is an example of this. The bending stress at the midship section section in this example takes more than sixty cycles and twenty seconds to decay to one-quarter of the maximum value measured at the slam. These stress reversals can be of considerable importance when considering fatigue and cyclic loading.

The machinery vibrations are often difficult to predict during the design stage and will be best treated by making adequate provisions for shock mounting and insulation. Flow-induced vibrations should be avoided as much as possible by careful hydrodynamic design.

* One ship life is normally taken to be equal to 20 ship years.

Figure 19. Sensitivity of On-Cushion, Design, Bending Moment To Selected Return Period and Weibull Parameter c.

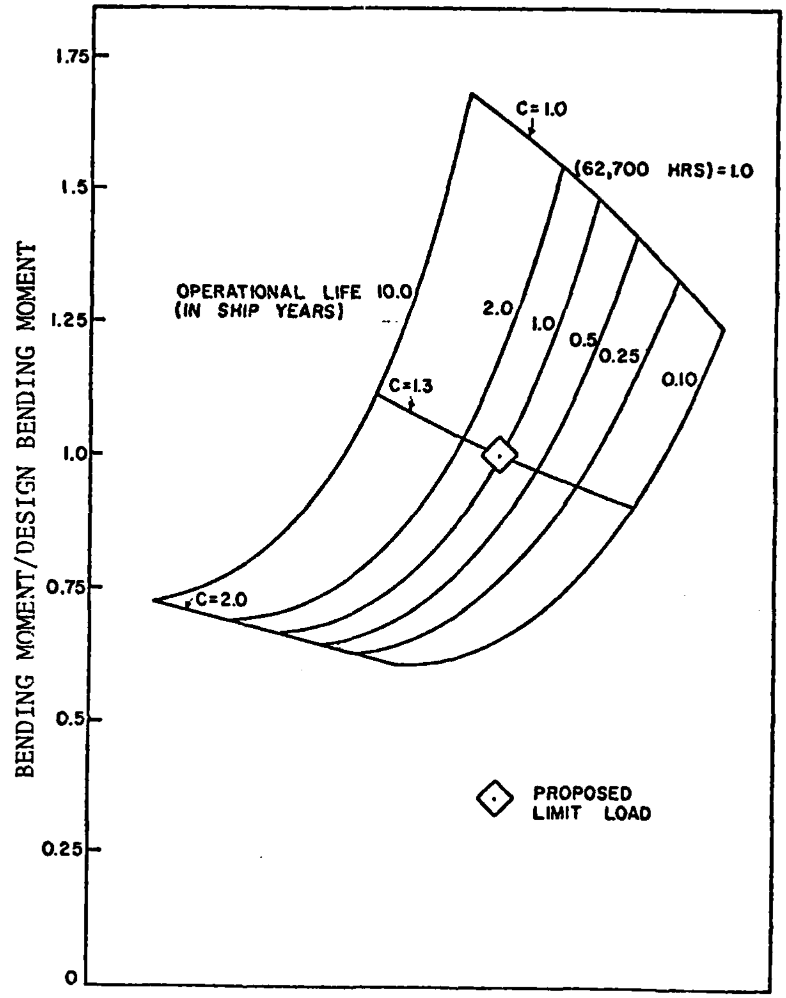
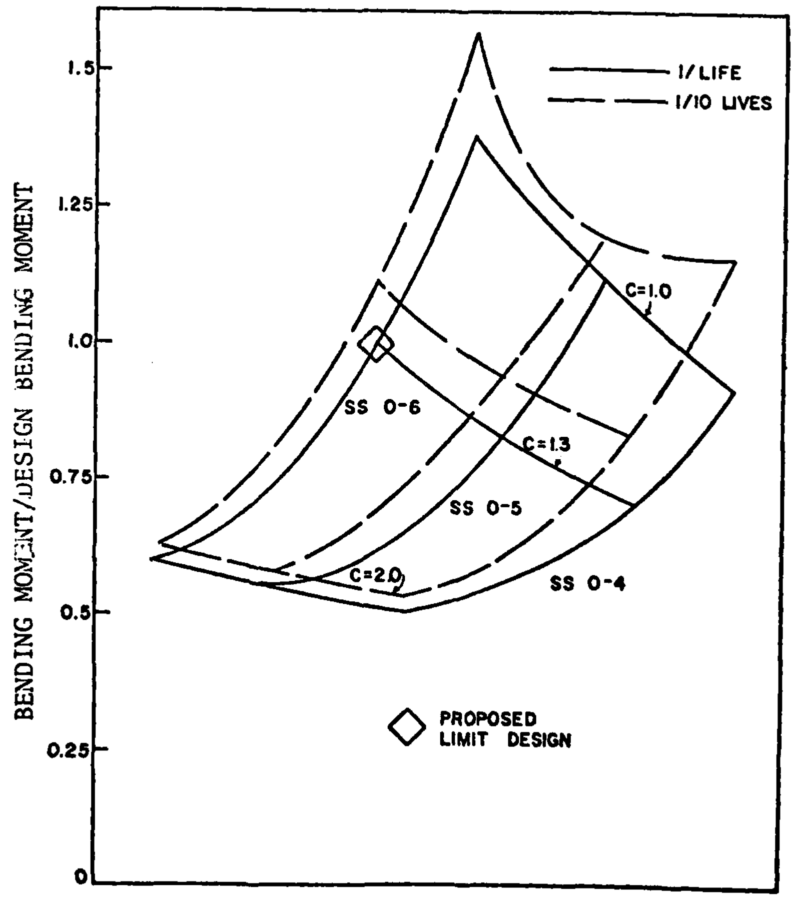


Figure 20. Sensitivity of On-Cushion Design Bending Moment to Limit On-Cushion Sea-State and Weibull Parameter c.



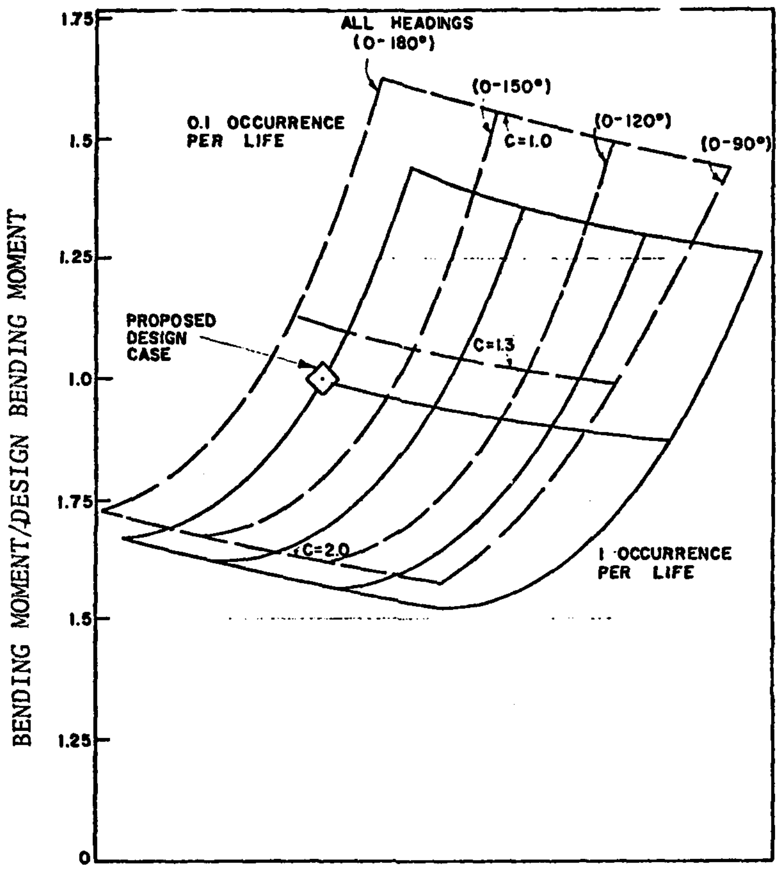


Figure 21. Sensitivity of On-Cushion, Design Bending Moment to Range of Heading Angle and Weibull Parameter c.

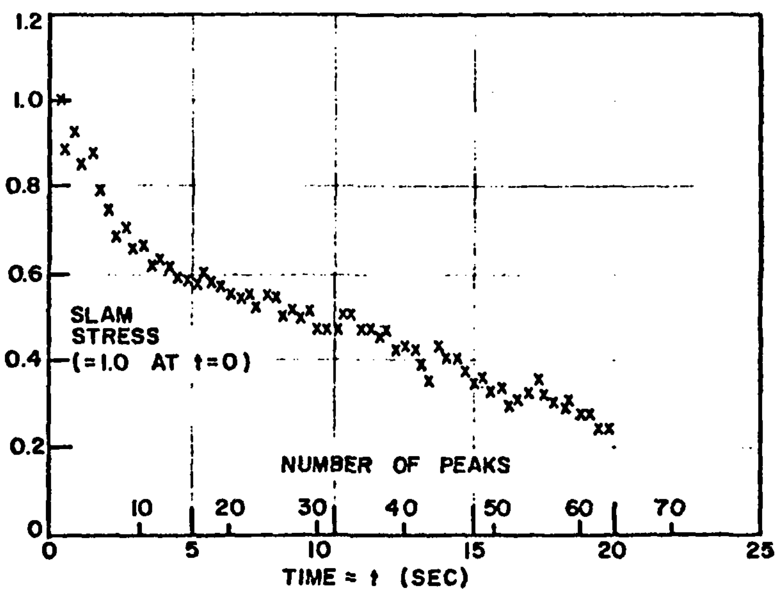


Figure 22. A Typical Decay Curve of Whipping Stress. (Reference 1).

The most critical of the cyclic loads are probably the wave-induced loads, as they occur with far greater frequency than the basically diurnal thermal effects or the load change cycles.

An illustration of the relative magnitudes and frequencies of the various types of loads acting on a conventional ship is shown in Figure 6. The stress record in this figure covers an entire voyage of a large tanker. Three types of load variation are apparent:

1. The vertical lines represent the wave bending stresses which were sampled for a comparatively short time every four hours. The higher frequency wave bending and vibrational loads cannot be properly represented on this scale. The ship evidently encountered a severe storm soon after leaving Perth Amboy.

2. Large changes are apparent in the mean stress level due to changes in ballast.

3. Superimposed on the mean time variation throughout the voyage are the daily variations due to temperature changes from day to night. These temperature effects are most apparent after passing around the Cape of Good Hope when the wave loads are very small. In the North Atlantic, when the wave loads are largest, the temperature effects are rather small.

An indication of the variation of severity of load with frequency of occurrence is shown in Figure 15. Again, the example shown is for a conventional ship. Each dot on the figure represents the root mean square of a sample of the hull bending stress on the S.S. HOOSIER STATE, taken every four hours during fourteen voyages. A large concentration of values are recorded at lower stresses and a very few at high stresses. By an extrapolation technique explained in Reference 25, it was possible to predict stress occurrences over a much longer time span. This result is shown in Figure 18, which plots the maximum stress against its probability of occurrence. Similar data for high performance ship impact loads can be computed analytically using the impact program described earlier. Figure 23 shows a result typical of these analyses. The right-hand end of the curve of loads occurring every few seconds provides data for fatigue and analysis, while the loads occurring at the left-hand end of the curve provide data for rarely occurring maximum loads.

The problem of achieving a proper design providing full protection against damage from the type of cyclic loading to which ships are exposed is a very serious one to which a great deal of attention is being paid, but no adequate analytical treatments are yet available.

The following approach should be adopted until sufficient stress data is available from some of the experimental high performance ships:

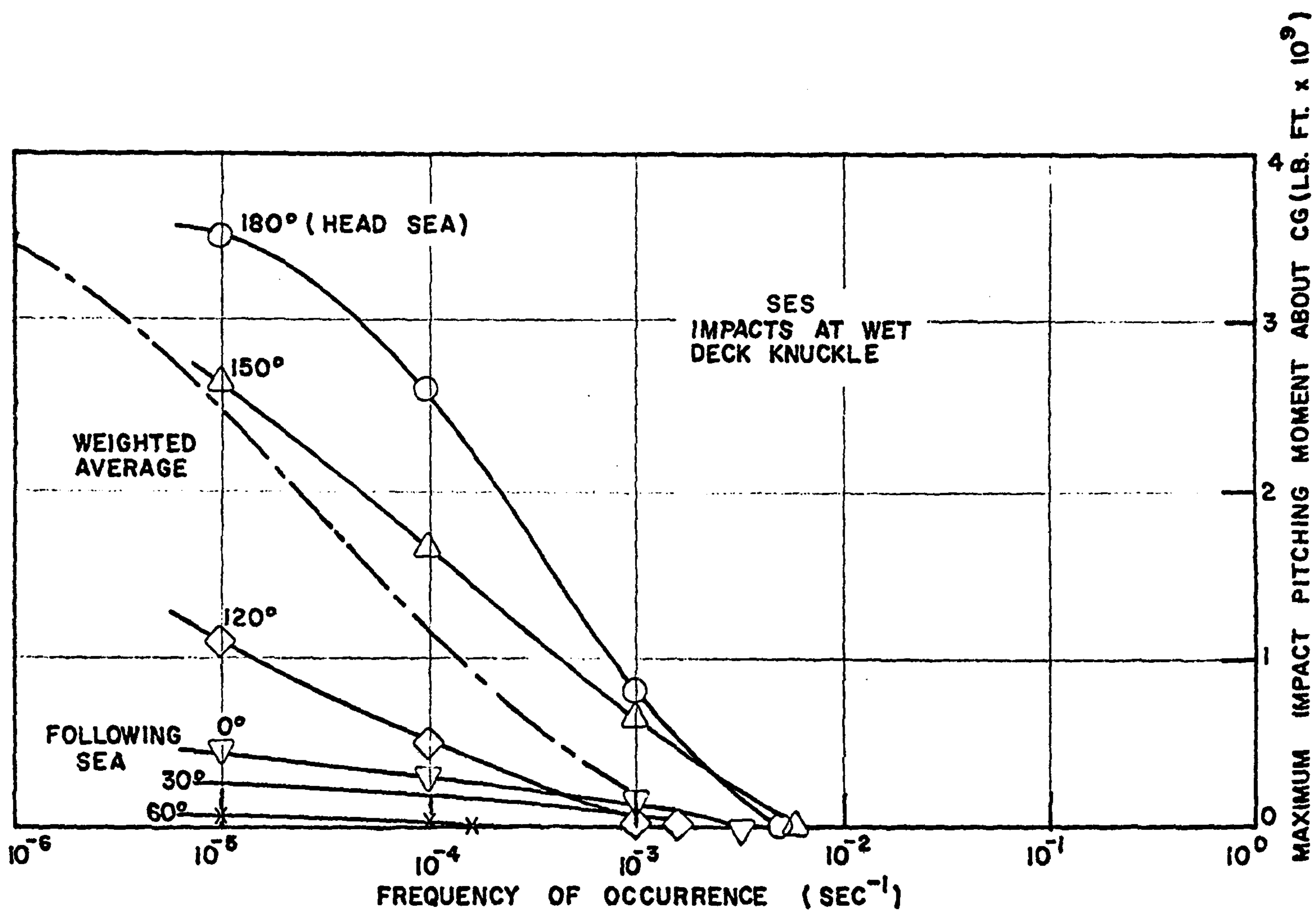


Figure 23. Frequencies of Occurrence of Pitching Moments During Impact.

1. Calculate the long-term probability distribution of stresses in critical components (main hull, girders, plating in impact areas, etc.) due to wave bending, slamming and thermal stresses. A series of curves similar to those in Figure 18 will then be obtained. When plotted against number of cycles, the long-term probability curve is usually referred to as the cyclic loading spectrum.

2. Experience with conventional ships suggests that the cyclic loading spectrum due to wave bending is slightly more severe than that due to whipping as described above. Figure 24 compares these two spectra for the case of the WOLVERINE STATE. The two spectra are nearly enough equal to suggest that the dynamic spectra may prove to be critical in a high performance vessel such as the SES, but until such information is available, it is proposed to assume that the dynamic effects are not more severe from considerations of cyclic loading than the wave bending effects which are more readily predictable.

Results of ACV and SES model tank tests reveal that in regular waves, a great deal of response is generated at frequencies other than the wave encounter frequency as a result of their inherently nonlinear response characteristics. The higher frequency components of these responses may well be significant for considerations of fatigue. The relative importance of the various harmonic components are sketched in Figure 25. Typical second harmonics can have an amplitude of more than one-half of the fundamental and may occur twice as frequently, so that the long-term probability curve is displaced to the right by a factor of two on Figure 25. Similarly, the third harmonic curve may be displaced to the right by a factor of three. By inspection of Figure 25. however, and by comparison with typical fatigue strength characteristics as sketched in Figure 26 (Reference 1), it does not appear that the harmonic components of the wave-induced bending loads can have a critical effect. The dominant characteristic of the wave-induced bending appears to be the maximum and minimum value. This simplifies calculation procedures provided that the model or full-scale data are available.

4. RECOMMENDED DESIGN CRITERIA

A typical approach to SES hull structural design and verification which can be adapted for any advanced marine vehicle is illustrated by the flow chart in Figure 27. Basically, two phases of the design development are envisioned. In the early stages of design the reliance is on criteria expressed in terms of sectional loads or overall load enveloped. Relatively simple analysis procedures can be employed, thereby allowing many conditions to be investigated. Approximately 15 to 30 maximum expected operating load (MEOL) cases are recommended. These loads should be presented as consistent sets at each station and are in the form of total integrated section forces, such as shear, bending moment and torsion, as well as local inertia and pressure loads. While the loads acting simultaneously at each section should be approximately correlated at that section, they are not necessarily correlated station-to-station.

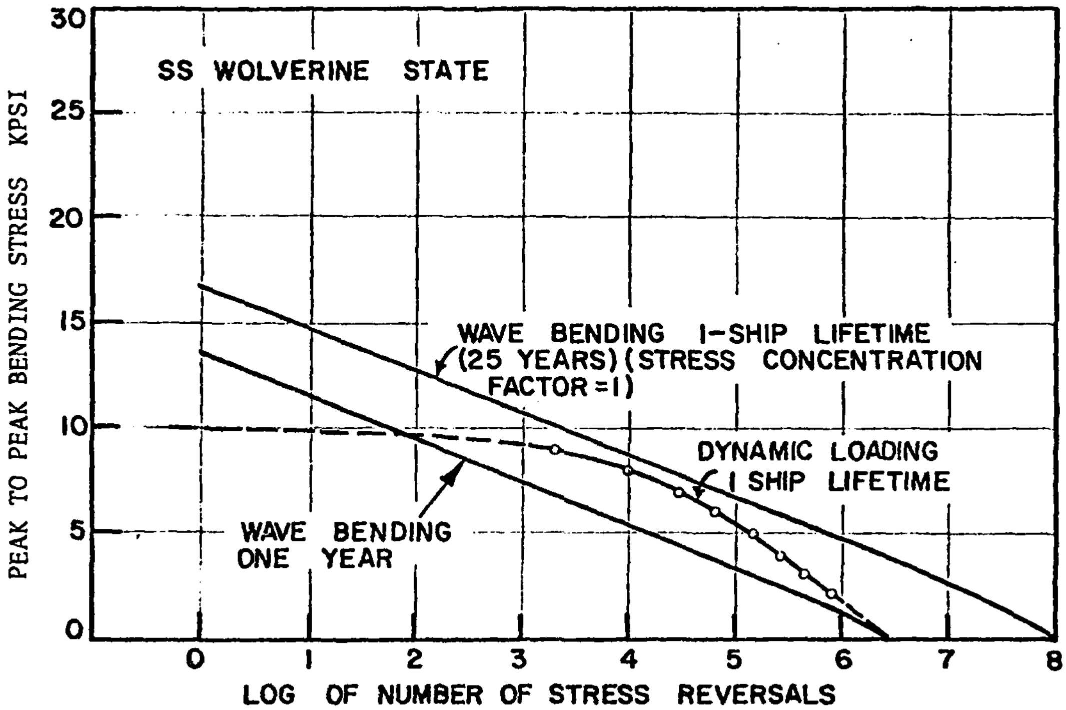


Figure 24. Cyclic Loading "Spectra". S.S. WOLVERINE STATE. (Reference 1)

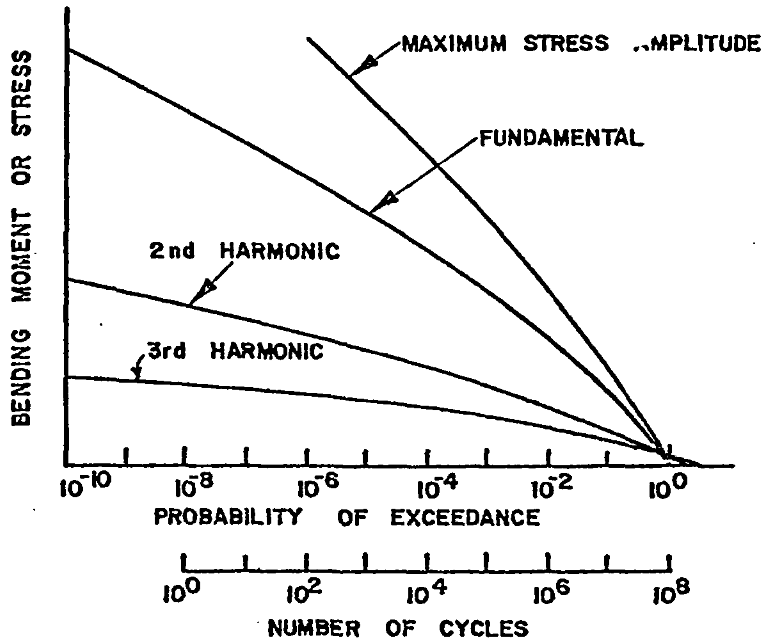


Figure 25. Cyclic Loading Spectra for Typical SES Model, Showing Harmonic Components of Total Stress.

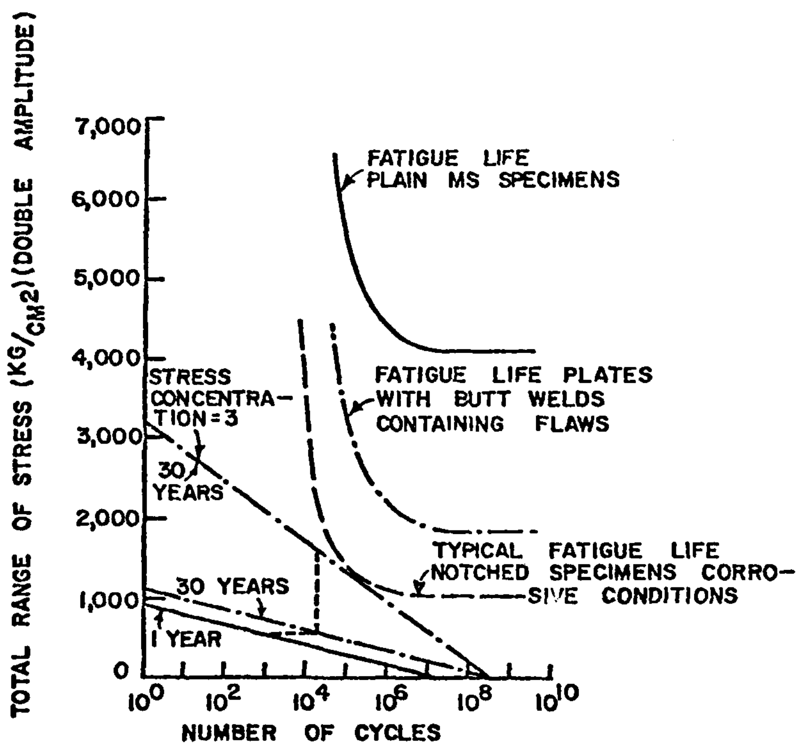


Figure 26. Example of Application of Cyclic Loading Curves to Study of Fatigue. (Reference 1).

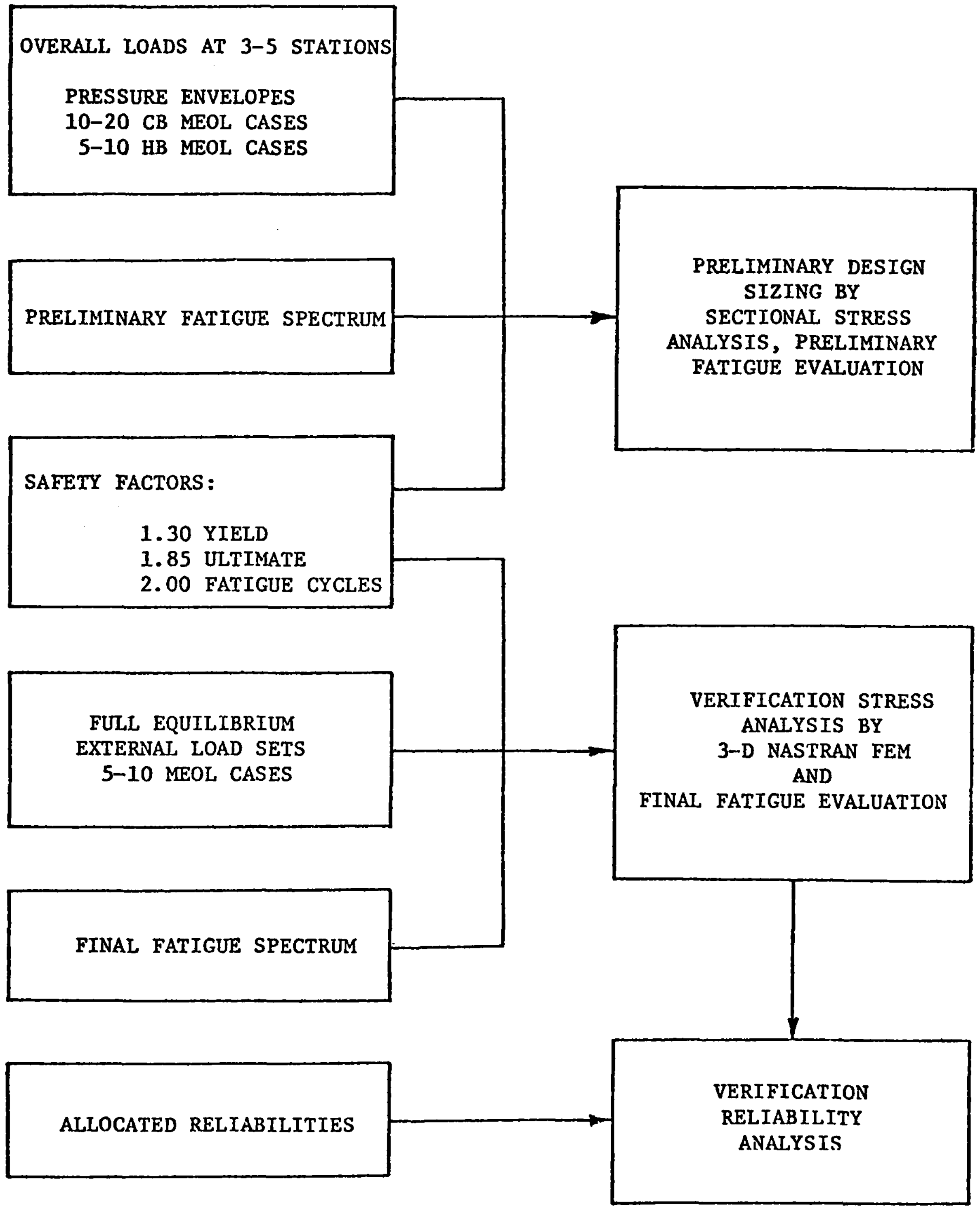


Figure 27. Typical General Approach to SES Hull Structural Analysis.

The stress analyses required during this first phase generally involve one and two-dimensional structural analyses wherein maximum stresses are checked against minimum design yeild and ultimate allowables. Minimum values of factors of safety are required which are presumed to ensure adequate reliability. A preliminary fatigue check is also recommended at this time, based on total lifetime loading cycles. A spread between predicted loading cycles and predicted cycles-to-failure, as specified by the fatigue safety factor, insures adequate fatigue reliability.

Once the structural configuration and scantlings are fairly well established, a verification phase of analysis is recommended, as shown in Figure 27. It is not anticipated completing this second cycle during the ANVCE Program point design but it is included here for completeness. Two complete verification checks are suggested. The first is a stress and fatigue cycle check based on full equilibrium loads. This check would involve a three-dimensional finite element model of the hull, where the input loads must be equilibrium sets of distributed pressures (i.e., cushion, buoyancy, hydrodynamic, aerodynamic and seal pressures), accelerations and other external forces (i.e., seal reactions, appendage forces and thrust loads). The criteria to be met is again based on the recommended minimum design safety factors.

The second verification check recommended is a complete lifetime reliability analysis to insure that the target hull reliability is met by the final design. This analysis is accomplished by the procedures illustrated in References 6 and 28.

Specific recommendations on load selection, safety factors and analysis methods are summarized in the following sections.

Load Selection Procedure

It is recommended that approximately 100 cushionborne and 50 hullborne cases, in the case of the SES, be analyzed by use of the linear frequency response programs. The conditions to be considered are defined by the operating envelope discussed earlier (see Figures 1, 4 and 5). The data from these 150 analyses, as well as still-water and thermal loads data, are then used to select about 15 to 30 extreme load cases. By interpolation and extrapolation, sufficient information to cover the entire operating envelope matrix can also be obtained, from which lifetime fatigue and MEOL loading spectra can be generated.

Each critical case selected is defined in terms of the maximum expected once-in-a-lifetime load which is based on the total time assigned to all speeds, sea states, headings and weights. A range of probable combinations of wave-induced, still-water and thermal loads are developed to represent the MEOL. The selected load cases are summarized in terms of sectional loads or load envelopes for use in the early stages of design, as mentioned above. The stress checks at this stage are made against the recommended minimum safety factors, which are defined below.

In the verification phase of the design, a relatively small number of critical MEOL design cases are selected based on the results of the preliminary design stress analysis. The MEOL design cases may then be narrowed down to about 5 to 10 cushionborne and 3 to 5 hullborne conditions. Furthermore, based upon the preliminary fatigue analysis, the number of critical regions of the hull is narrowed down to 3 to 5 regions or components. Detail loads analyses are then performed which could involve time-domain analyses of motions, loads and impacts as well as further, frequency-domain analyses and hull-dynamic-response analyses. The purpose of these analyses is to obtain full-equilibrium, external load sets for use in verification stress analysis by 3-D, NASTRAN, finite-element models. In addition, finalized, fatigue-loading spectra are obtained for the most critical regions. The verification stress analysis insures that the design meets the recommended minimum safety factors.

A final substantiation of the hull design is recommended through a reliability analysis wherein allocated reliabilities are checked against assessed reliabilities by the procedures as developed, for example, in References 6 and 28. Reliability analysis procedure is beyond the scope of the present effort but a suitable procedure is outlined in Reference 28.

Design Safety Factors

It is recommended that the safety factors specified for SES hull design, as shown in Table 2, be used for all vehicles in the ANVCE study. The recommended yield and ultimate safety factors follow the conventional definition.

$$S.F. = \frac{\text{Design Minimum Strength}}{\text{Mean MEOL Stress}} \tag{4}$$

Here, the "minimum" strength is defined as the 99 percent probability, 95 percent confidence value, based on one-sided tolerance limits of the normal distribution. The mean MEOL stress is based on the highest loads that occur, on the average, once in a ship lifetime.

In the case of the SES, ACV and hydrofoil, the recommended factors of safety are the same for hullborne and foilborne or cushionborne operational conditions.

The recommended fatigue safety factor of 2.00 is in terms of cycles. Thus, for example, an SES hull or waterjet inlet should not fail in fatigue for two lifetimes. This criteria may be found to be too severe for the full ship lifetime of 20 years, in which case periodic repairs and/or replacement of certain critical structural components would have to be scheduled. The fatigue safety factor for such components would then apply to the loading cycles expected to occur between scheduled repair periods.

Table 2. Recommended Safety Factors.

PARAMETER	NORMAL OPERATION	EMERGENCY OPERATION
Design Yield Stress	1.30	1.00
Design Ultimate Stress	1.85	1.50
Fatigue Cycles	2.00	N/A

5. SPECIAL CONSIDERATIONS FOR ANVCE CANDIDATE VEHICLES

Conventional Ships

Critical structural loads for conventional ships are normally wave-induced bending loads. In some smaller, high-speed ships bottom slamming may be a problem and in very large vessels the wave-induced springing loads provide the design loads.

Surface Effect Ships

The main hull of the SES structure is usually designed by the combined wave-induced and slamming loads experienced during operation in high sea states. Both the on-cushion, high-speed and the low speed, off-cushion cases should be examined.

Air Cushion Vehicles

Air cushion vehicles are very similar, structurally, to the SES. Design load conditions may be caused by slamming or plow-in cases. For amphibious ACV's the design loads are frequently those due to standing at rest on land.

Hydrofoils

The hull structure of the hydrofoil is usually designed by foilborne operating condition in waves. The probability of and severity of hull/wave impact should also be examined.

SWATH

The SWATH ship is subjected to the transverse bending and torsional loads experienced by catamarans but the excursions of these loads about their mean values should be relatively small due to the relatively small response of the SWATH hull to surface waves. Slamming load magnitudes and probabilities should be computed.

Planing Hulls

Planing hulls must be designed to withstand the pressures and loads caused by pounding in severe sea states. Supercritical hulls which have demonstrably lower accelerations in waves may be designed to correspondingly lower loadings.

Wing-in-Ground Effect (WIG)

The WIG may be designed, structurally, as a conventional aircraft, taking due account of the very much reduced maneuverability envelope, except for the problem of inadvertent contact with waves. The probability of wave impact should be computed and appropriate impact cases should be included in the design.

Sea-Loiter Seaplanes

Seaplanes designed for the sea-loiter mission must be designed to withstand exposure to any sea state, in the loiter mode, unless it can be demonstrated that it has adequate range capability to remove it from any storm area.

Other Aircraft

Other aircraft should be designed to established aircraft design procedures with due allowance made for the particular requirements of each mission profile.

REFERENCES

1. Lewis, E.V., Hoffman, D., Maclean, W.M., van Hoof, R. and Zubaly, R.B. "Load Criteria for Ship Structural Design", SSC-240, (1973).

2. St. Denis, M. & Pierson, W.J. "On the Motions of Ships in Confused Seas", Trans. of SNAME, Vol. 61, (1953).

3. Lewis, E.V. "Predicting Long-Term Distributions of Wave-Induced Bending Moment on Ship Hulls", No. 6, SNAME, Spring Meeting, Montreal, (1967).

4. MIL-A-8629(Aer), Military Specification - Airplane Strength and Rigidity, (Aug. 28, 1953).

5. Lavis, D.R., Bartholomew, R.J. & Jones, J.C. "Response of Air Cushion Vehicles to Random Seaways and the Inherent Distortion in Scale Models", AIAA Paper No. 72-598, (July 1972).

6. "Structural Design Criteria Extreme Value - Loads and Safety Factors Criteria Analysis Report", Rohr Industries, Inc., Doc. No. DH11S00505, ID No. AP-003923, Contract No. N00024-74-C-0924, (Dec. 19, 1975).

7. "Loads Criteria Model Tests - Test Data Analysis and Correlation Report", Rohr Industries, Inc., Doc. No. DH10S00G02, ID No. AP-004114, Contract No. N00024-74-C-0924, (Jan. 31, 1976).

8. Kaplan, P., Benton, J. & Sargent, T.P. "A Study of SES Loads and Motions", Oceanics Report No. 71-84, (August 1971).

9. Band, E.G.U. "Study of Bow Impact Loads for 100 Ton Surface Effect Ship", Wyle Laboratories, Payne Division, Working Paper No. 18008-2, (11 December 1969).

10. Band, E.G.U. "Calculation of Cushion Borne Bow Impact Loads on the JEFF-A Landing Craft", for Aerojet General Corp., Report No. EB03, (August 1971).

11. Band, E.G.U. "The Bow Impact Problem for Arctic Surface Effect Vehicles", Payne, Inc. Working Paper No. 104-10, (May 1973).

REFERENCES (Con't.)

12. Pierson, J.D. "The Penetration of a Fluid Surface by a Wedge", Stevens Institute of Technology, Experimental Towing-Tank Report No. 381, (July 1950).

13. Pierson, J.D., Dingee, D.A., & Neiding, J.W. "A Hydrodynamic Study of the Chines-Dry Planing Body", Stevens Institute of Technology, Experimental Towing Tank Report No. 492, (May 1954).

14. Korvin Kroukovsky, B.V. & Chabrow, F.R. "The Discontinuous Fluid Flow Past an Immersed Wedge", Stevens Institute of Technology, Experimental Towing Tank Report No. 334, (October 1948).

15. King, D.A. and Adams, R.C. "Martin Model 270 Water Loads Investigation, Hull Bottom Pressures and Impact Loads", Martin Co. Engineering Report ER 7516, (November 1955).

16. "Loads Criteria Structural Loads Model (Segmented Centerbody) Tow Tank & Impact Test Report", Rohr Industries, Inc., ID No. AP2-003454, Doc. No. DH10S00F02, (26 September 1975).

17. Shafer, W.R. "The Feasibility of Fabricating and Testing a 1/10-Scale Rigid Vinyl Model of the Bell Aerospace SES-100 Captured Air Bubble Vehicle", NSRDC n 176, (November 1970).

18. Austin, S.L. "Rigid Vinyl Modeling: An Efficient Technique for Predicting Ship Structural Response", paper presented to Second Ship Structures Workshop, NSRDC, (Feb. 1973).

19. "Loads Criteria Structural Loads Model (Flexible Centerbody) Oblique Sea Test Report", Rohr Industries, Inc. ID No. AP2-003658, Doc. No. DH10S00F03, Contract No. N00024-74-C-0924, (7 November 1975).

20. "Loads Criteria Structural Loads Model (Flexible Centerbody) Tow Tank & Impact Test Report", Rohr Industries, Inc. ID No. AP2-003999, Doc. No. DH10S00F05, Contract No. N00024-74-C-0924, (23 Dec. 1975)

REFERENCES (Con't.)

21. "SES-100A Program Status Report", Aerojet General Corporation Report No. AGC-T-409, (Sept. 17, 1973).

22. "SES-100B Test and Evaluation Final Report", Bell Aerospace Co. N. 7308-948010, CONFIDENTIAL, (Sept. 17, 1973).

23. Saunders, H.E. Hydrodynamics in Ship Design, Pub. by SNAME, New York, New York, (1965).

24. Pierson, W.J., Jr., Neumann, G. & James, R.W. Practical Methods for Observing and Forcasting Ocean Waves by Means of Wave Spectra and Statistics, Pub. by U.S. Naval Oceanographic Office, H.O. Pub. No. 603, (1955).

25. Band, E.G.U. "Analysis of Ship Data To Predict Long-Term Trends of Hull Bending Moments", American Bureau of Shipping, New York, (1966).

26. Gumbel, E.J. "Statistical Theory of Extreme Values and Some Practical Applications", National Bureau of Standards, Applied Mathematics Series No. 33, (February 1954).

27. Nibbering, J.J.W. "Fatigue of Ship Structures", International Shipbuilding Progress, (Sept. 1963).

28. "Criteria Applications and Structural Design Criteria Final Summary Report", Rohr Industries, Inc., ID No. AP2-004213, Doc. No. DH11S005,701, Contract No. N00024-74-C-0924, (27 February 1976).

APPENDIX A

CHARACTERISTICS OF THE WEIBULL DISTRIBUTION

The generalized gamma distribution is described by the probability density function

$$f(x) = \frac{c}{\Gamma(m)} \lambda^{cm} x^{cm-1} \exp[-(\lambda x)^c] \tag{A-1}$$

where c, λ and m are the three parameters of the distribution.

The cumulative density function is given by

$$Q(x) = \int_0^x f(x)\,dx \tag{A-2}$$

The mean value of the distribution is given by

$$\bar{x} = \int_0^\infty f(x) \cdot x\,dx = \frac{\Gamma(m + 1/c)}{\Gamma(m)\lambda} \tag{A-3}$$

The standard deviation σ about the mean value is given by

$$\sigma = \left\{ \int_0^\infty f(x)(x - \bar{x})^2\,dx \right\}^{1/2}$$

$$= \left\{ \frac{\Gamma(m + 2/c)}{\Gamma(m)\lambda^2} - \bar{x}^2 \right\}^{1/2} \quad \text{and} \tag{A-4}$$

$$\sigma\lambda = \left\{ \frac{\Gamma(m + 2/c)}{\Gamma(m)} - (\bar{x}\lambda)^2 \right\}^{1/2}$$

The root mean square value about the $x = o$ axis is given by

$$x_{rms} = \left\{ \int_0^\infty f(x)\, x^2\, dx \right\}^{1/2}$$

$$= \left\{ \frac{\Gamma(m + 2/c)}{\Gamma(m)\lambda^2} \right\}^{1/2} \quad \text{and} \qquad \text{(A-5)}$$

$$x_{rms}\lambda = \left\{ \frac{\Gamma(m + 2/c)}{\Gamma(m)} \right\}^{1/2}$$

Note that $x_{rms} = (\sigma^2 + \bar{x}^2)^{1/2}$.

The values of $\lambda\bar{x}$, $\lambda\sigma$ and λx_{rms} are plotted for a range of values of c and m in Figure A-1.

The generalized gamma distribution reduces to the Weibull distribution in the special case when $m = 1$ and the Weibull distribution reduces to the exponential distribution when $c = 1$ and to the Rayleigh distribution when $c = 2$. These three distributions are indicated on Figure A-1.

The Weibull probability density function is obtained by substituting $m = 1$ in equation (A-1) which leaves λ and c as the two parameters:

$$f(x) = c\lambda^c\, x^{c-1} \exp[-(\lambda x)^c] \qquad \text{(A-6)}$$

The mean, mode, median and rms values are given by

$$\lambda\bar{x} = \Gamma(1 + 1/c)$$

$$\lambda x_{mode} = [(c-1)/c]^{1/c}$$

$$\lambda x_{median} = [-\ln(\tfrac{1}{2})]^{1/c}$$

and

$$\lambda x_{rms} = [\Gamma(1+2/c)]^{1/2}$$

from equations (A-3) and (A-5).

The cumulative probability function is given by

$$P(x) = 1 - Q(x) = \int_x^\infty f(x)\, dx$$

$$= \exp[-(\lambda x)^c] \qquad \text{(A-7)}$$

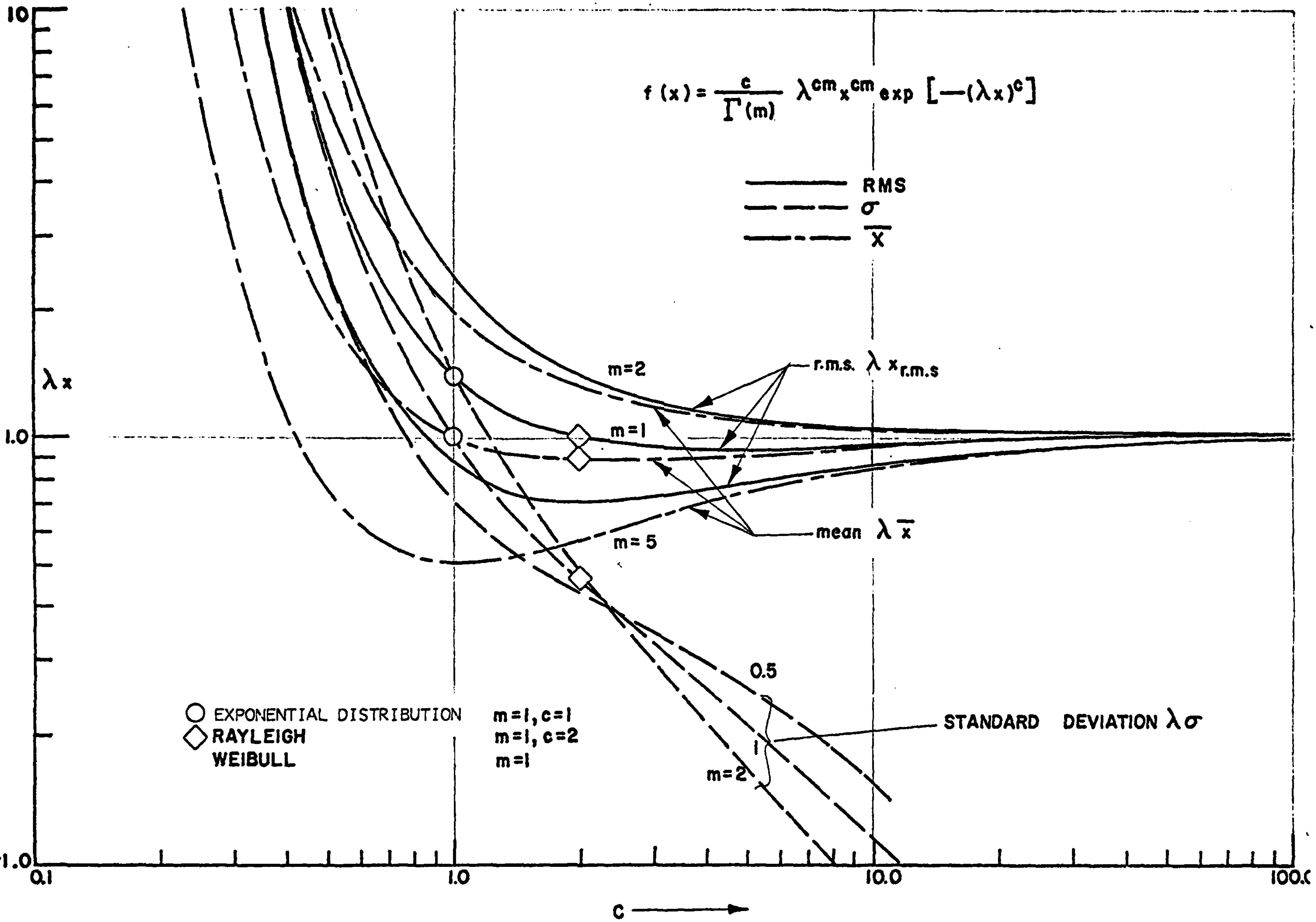


Figure A-1. Characteristics of the Generalized Gamma Distribution.

$f(x)$ and $P(x)$ are plotted for a range of values of c in Figures A-2, A-3 and A-4. Other terms that are often used to describe statistical distributions are the "average of the highest one-third, $\overline{x}_{1/3}$" and the "average of the highest one-tenth, $\overline{x}_{1/10}$" etc. In general, the "average of the $\overline{x}_{1/b}$" values as defined in Figure A-5 can be evaluated as follows.

The value, $x_{1/b}$, above which 1/b of the values of x are expected to fall is given by putting $P(x) = 1/b$ in equation (A-7) and solving for $x_{1/b}$:

$$\frac{1}{b} = \exp\left[-(\lambda x_{1/b})^c\right] \tag{A-8}$$

i.e. $\lambda x_{1/b} = [-\ln(1/b)]^{1/c}$

The average $\lambda\overline{x}_{1/b}$ of the highest 1/b values is therefore given by

$$\lambda\overline{x}_{1/b} = \frac{\int_{x_{1/b}}^{\infty} x \cdot f(x)\, dx}{\int_{x_{1/b}}^{\infty} f(x)\, dx} = b\int_{x_{1/b}}^{\infty} c\lambda^c x^c \exp\left[-(\lambda x)^c\right] dx \tag{A-9}$$

This can be evaluated numerically. The probability $P(\overline{x}_{1/b})$ of exceeding the value $\overline{x}_{1/b}$ is given by:

$$P(\overline{x}_{1/b}) = \exp\left[-(\lambda\overline{x}_{1/b})^c\right] \tag{A-10}$$

which can also be evaluated numerically.

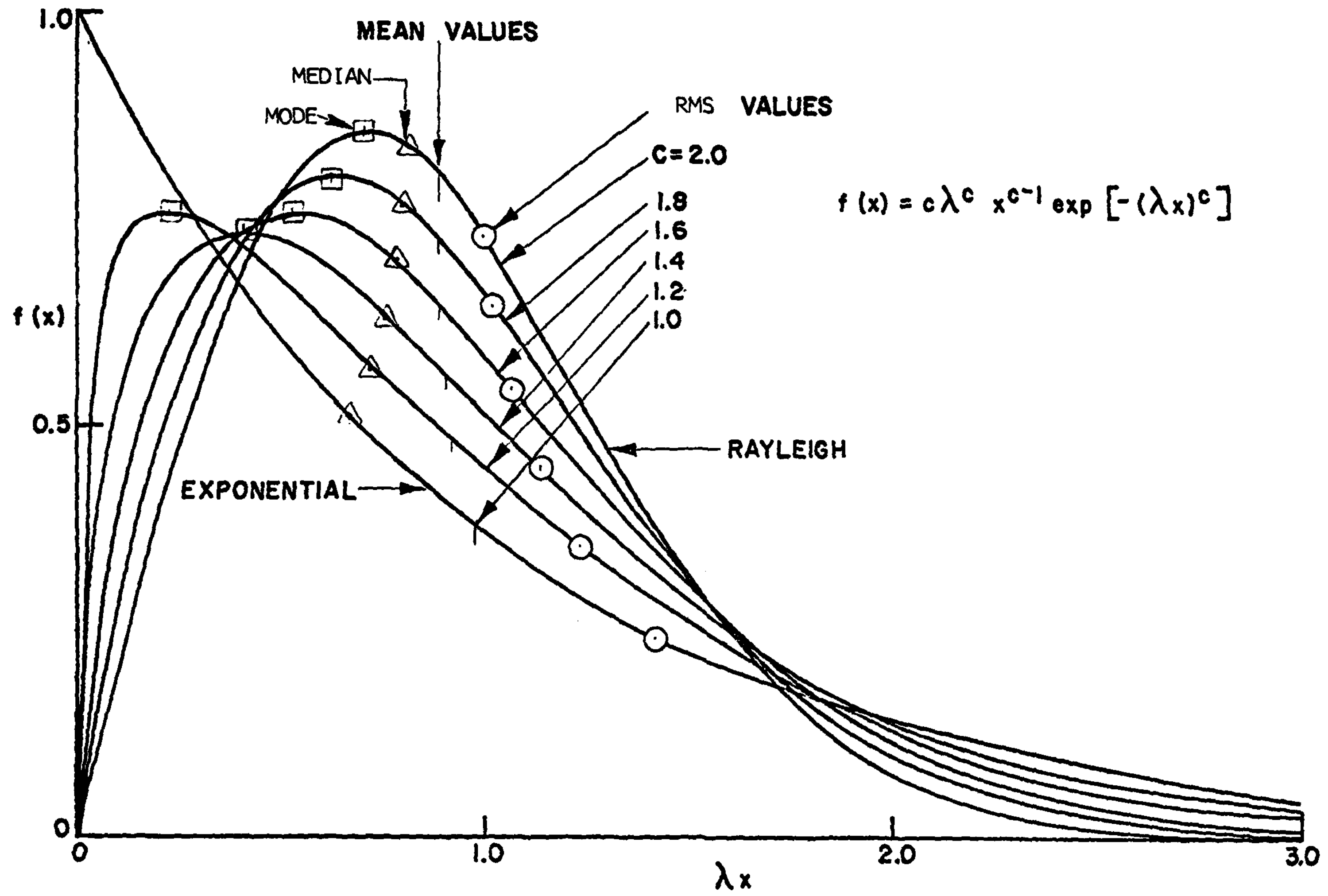


Figure A-2. The Weibull Probability Density Function.

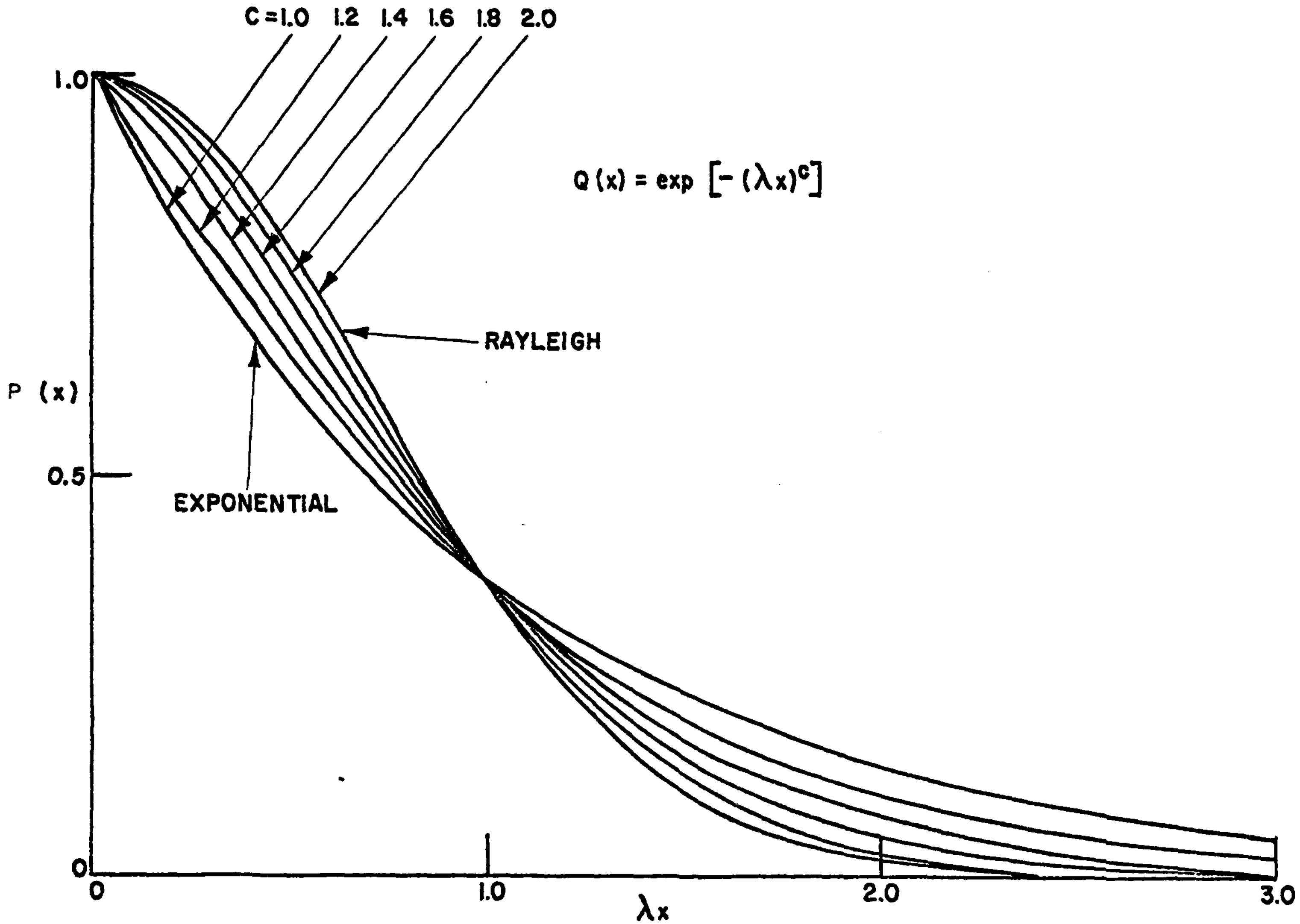


Figure A-3. The Weibull Cumulative Probability for a Range of Values of c.

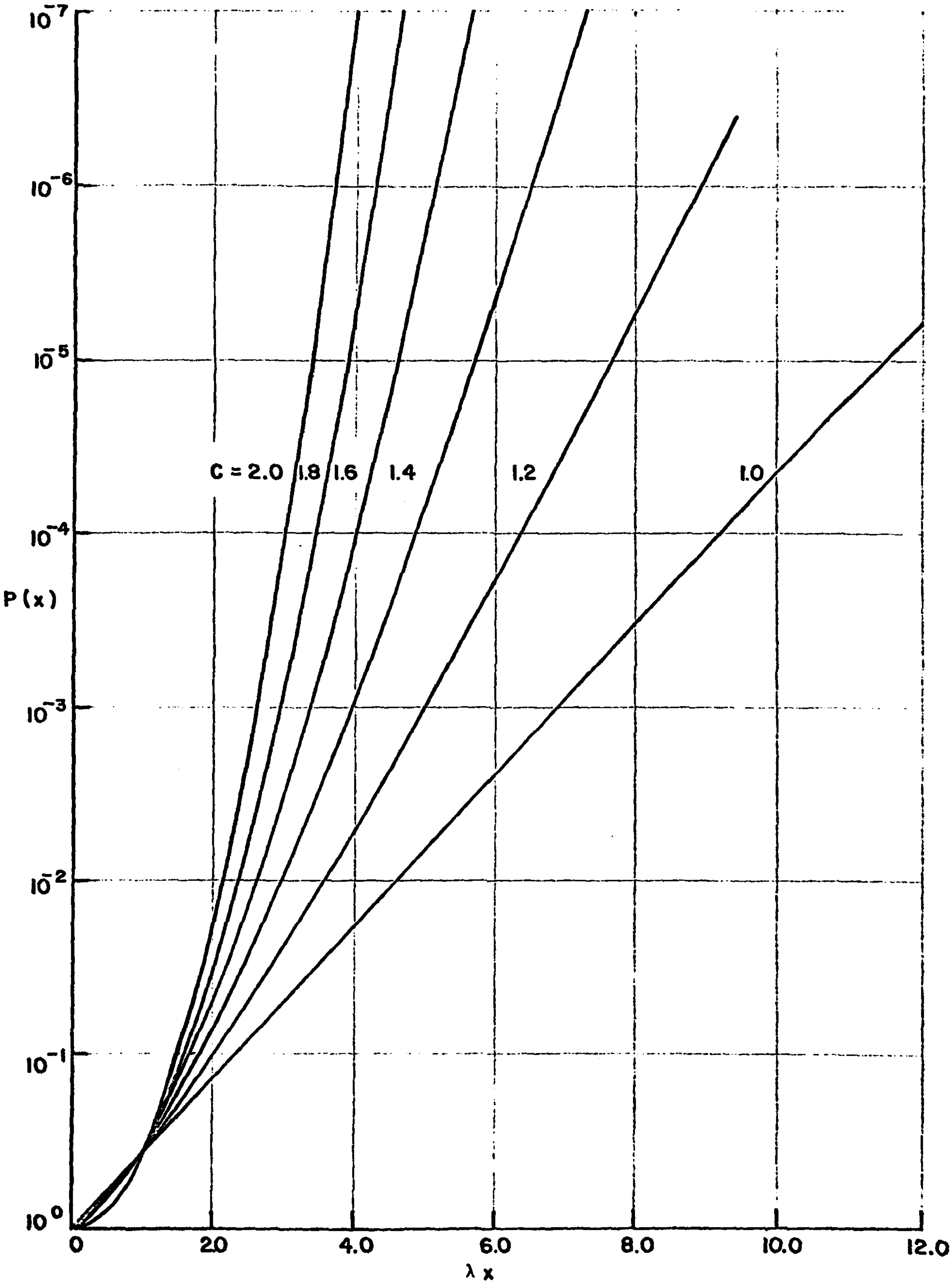


Figure A-4. The Weibull Cumulative Probability Plotted on Semi-Log Graph Paper.

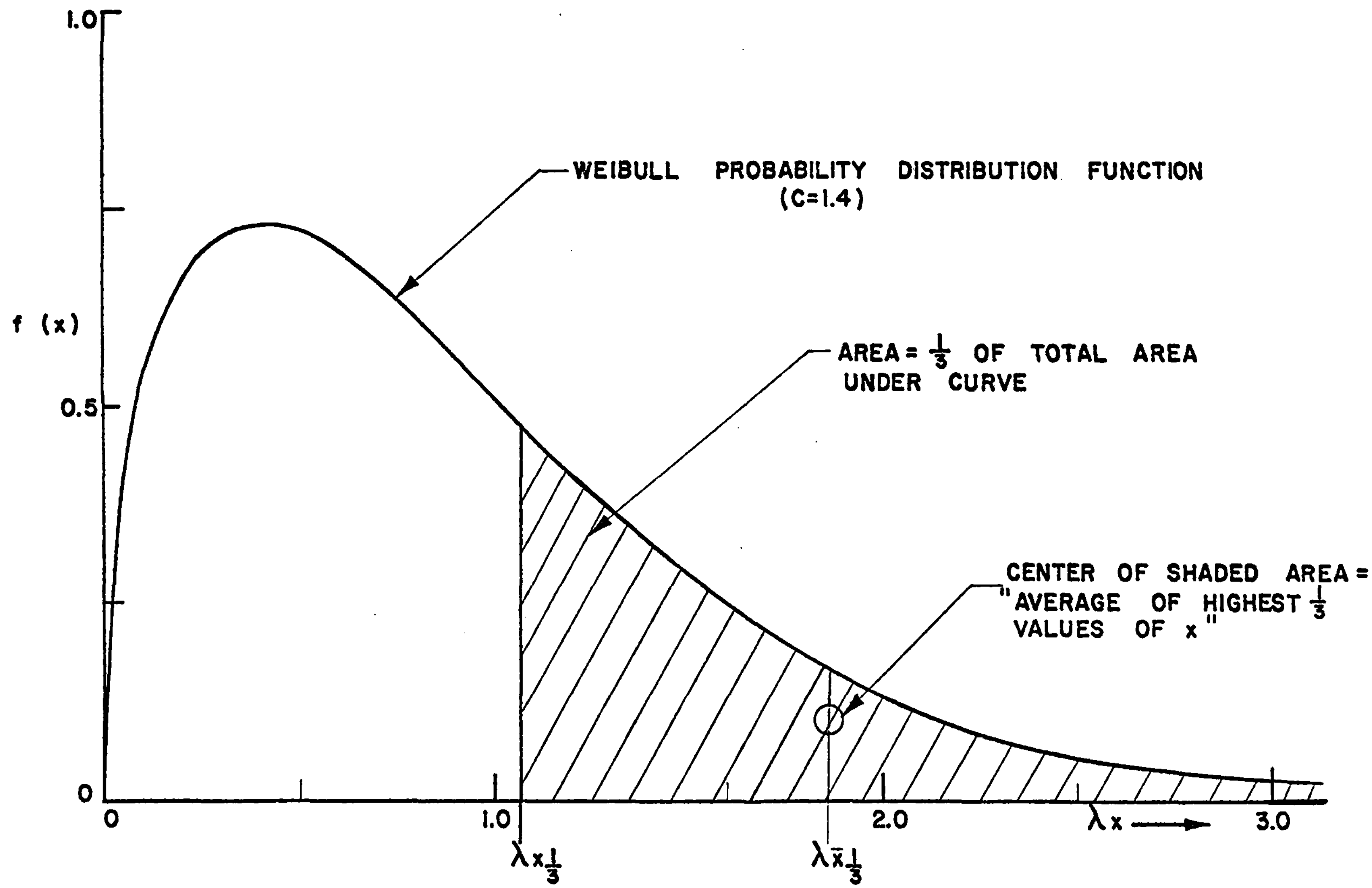


Figure A-5. Definition of "Average of Highest 1/b Values of x".

Values of $\tilde{x}_{1/b}$ and $P(\tilde{x}_{1/b})$ are plotted in Figures A-6 and A-7, respectively. In Figure A-7, it may be noted that the average of the highest 1/3 and 1/10 values do not vary greatly from one value of c to another. The range of variation of the mean values is larger.

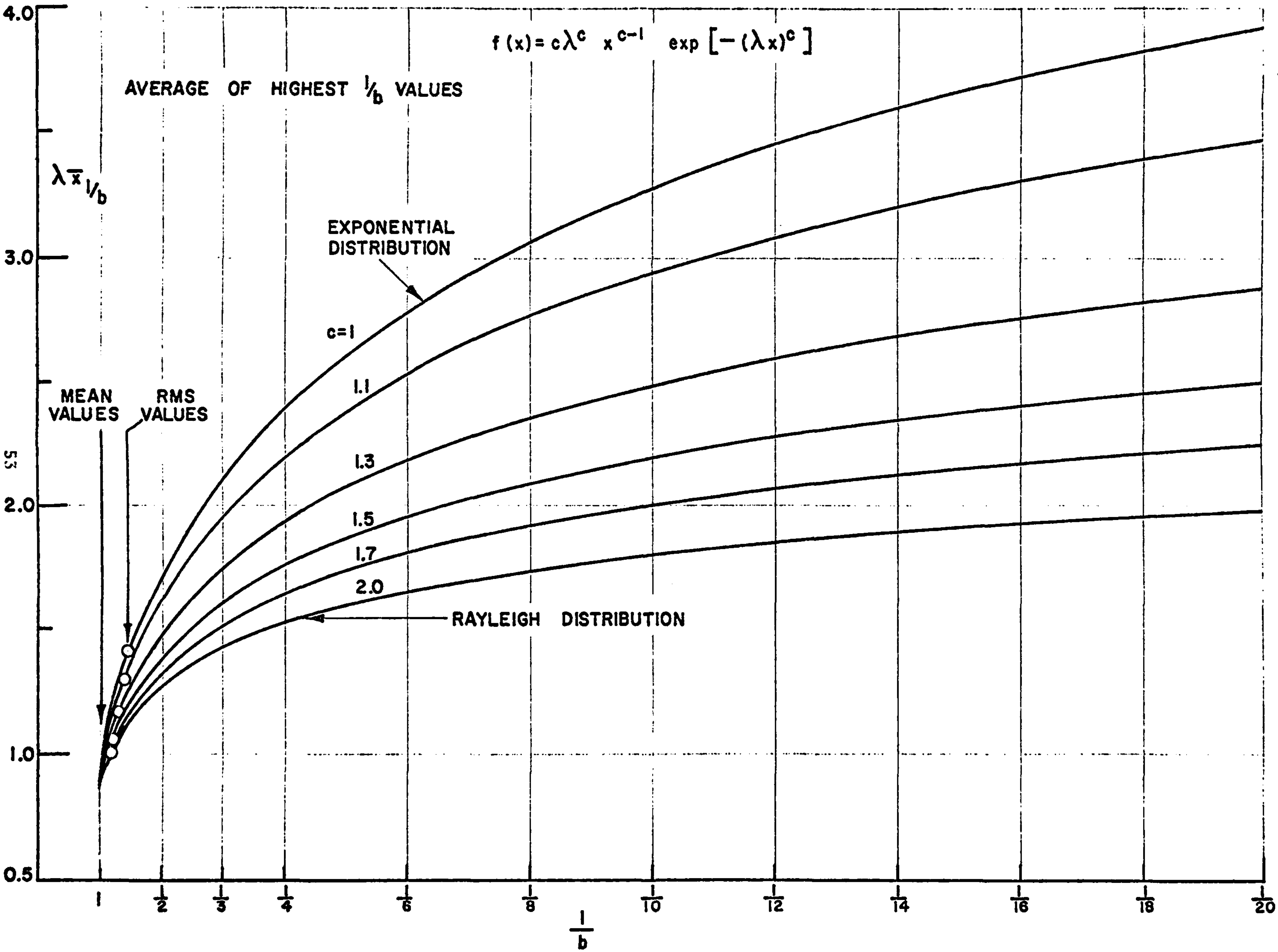


Figure A-6. Characteristics of the Weibull Distribution.

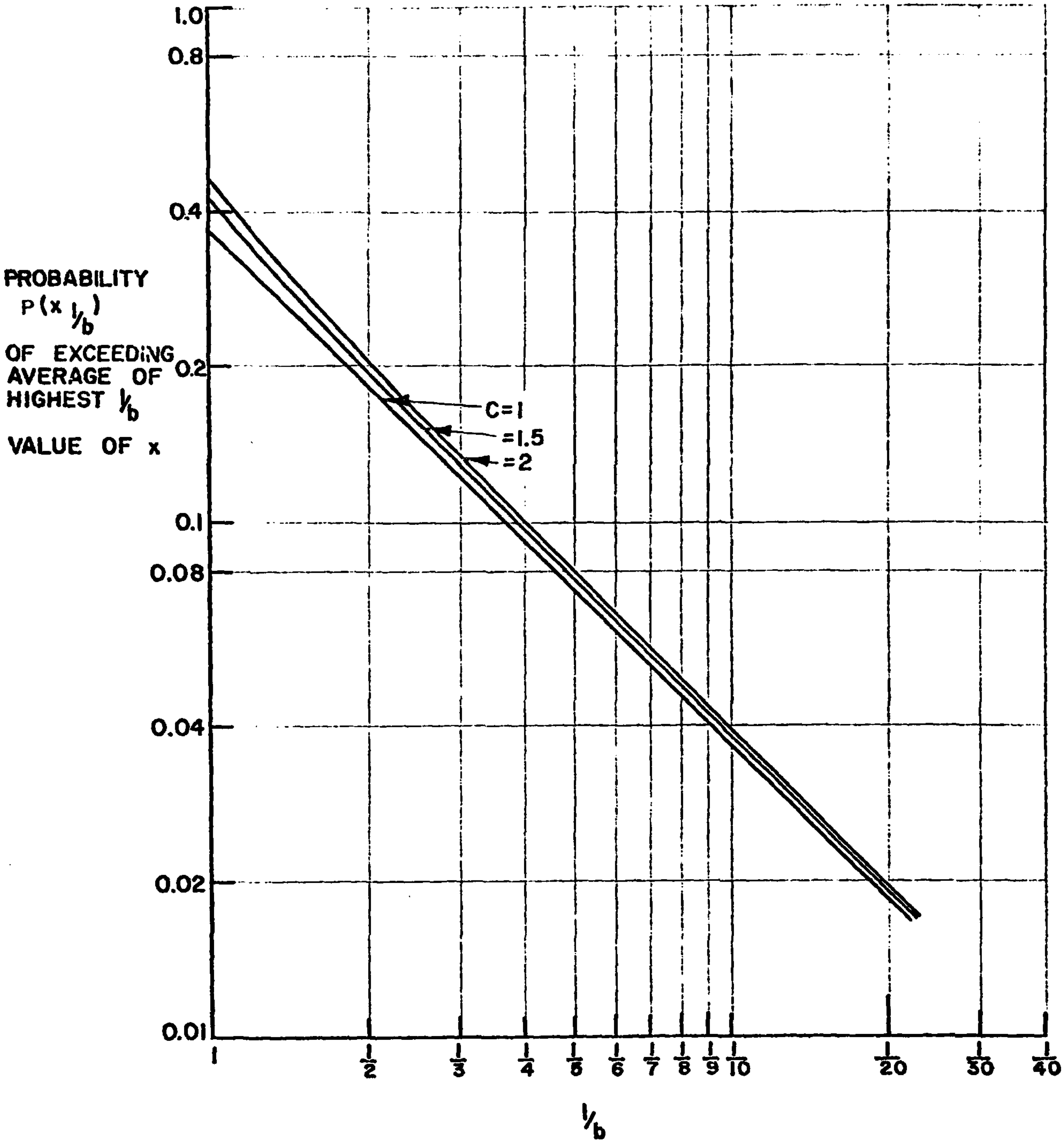


Figure A-7. Characteristics of the Weibull Distribution.